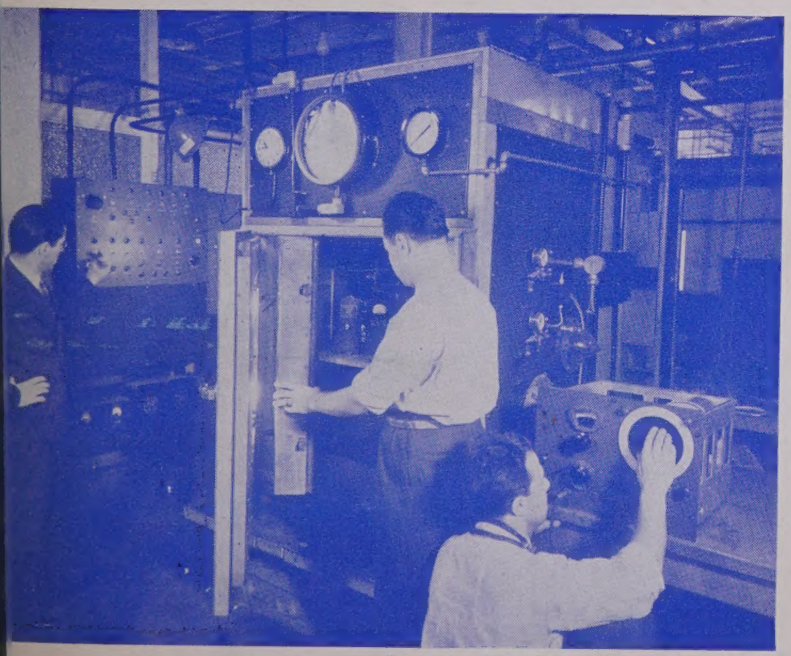
Proceedings of the I·R·E



International Telephone and Telegraph Corporation

SIMULATED DIVE-BOMBING TEST
Controlled cycling of pressure, temperature, and humidity enables radio-component test

JANUARY, 1944

VOLUME 32 NUMBER 1

Electron Microscope Optics

Bandwidths in Video Amplifiers

Quarter-Wavelength Line

Receiving Antenna

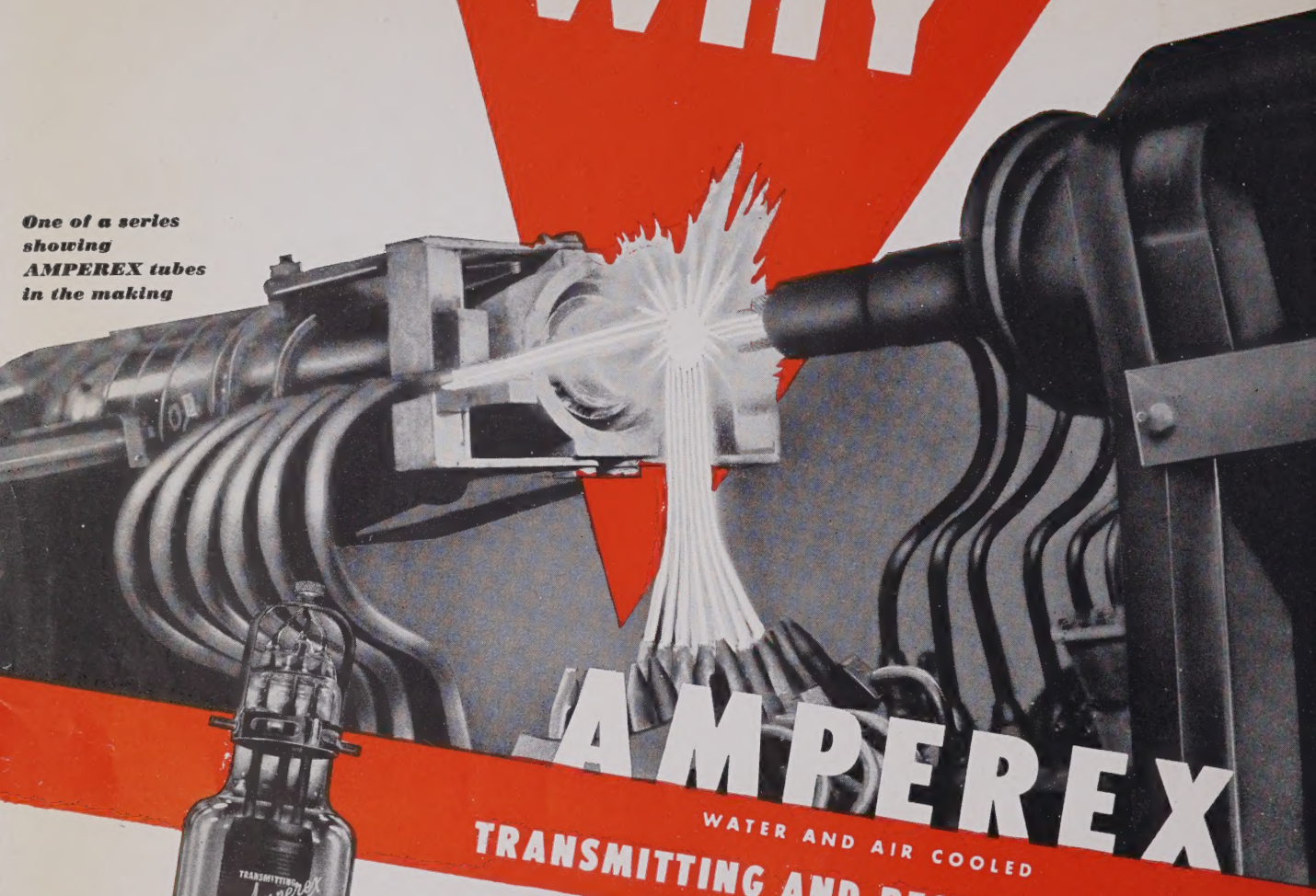
Receiving Antenna in Polarized Field

WINTER TECHNICAL MEETING

NEW YORK, N. Y., JANUARY 28 AND 29, 1944

Institute of Radio Engineers

WHY



One of a series showing AMPEREX tubes in the making

AMPEREX

WATER AND AIR COOLED

TRANSMITTING AND RECTIFYING TUBES

AMPEREX ... the high performance tube

Amperex "key" men have been associated with the vacuum tube art ever since its inception. Working against a background unique in the field, our engineers and production people are given free rein to conduct independent research and experimentation. Unhampered by mass production limitations, their high standards have resulted in advanced designs, greater efficiency, lower cost and longer life. Such "Amperextras" have carried our tubes to a commanding position in communications, ultra high frequency transmission, electro medical apparatus, high voltage rectification and many industrial applications.

ARMY E NAVY

START THE NEW YEAR WITH EXTRA PURCHASES OF WAR BONDS

AMPEREX ELECTRONIC PRODUCTS

79 WASHINGTON STREET BROOKLYN 1, N. Y.

BOARD OF DIRECTORS
1943
Lynde P. Wheeler, *President*
F. Sherbrooke Barton, *Vice President*
Raymond A. Heising, *Treasurer*
Haraden Pratt, *Secretary*
Alfred N. Goldsmith, *Editor*
Stuart L. Bailey
Wilmer L. Barrow
E. Finley Carter
Adolph B. Chamberlain
Ivan S. Coggeshall
William L. Everitt
Harold T. Friis
Gilbert E. Gustafson
O. B. Hanson
Frederick B. Llewellyn
Frederick E. Terman
Browder J. Thompson
Hubert M. Turner
Arthur F. Van Dyck
Harold A. Wheeler
William C. White

•

Harold R. Zeamans, *General Counsel*

•

BOARD OF EDITORS
Alfred N. Goldsmith, *Editor*
Ralph R. Batcher
Philip S. Carter
Lewis M. Clement
John F. Dreyer, Jr.
Elmer W. Engstrom
William L. Everitt
Peter C. Goldmark
Frederick W. Grover
C. M. Jansky, Jr.
John D. Kraus
Frederick B. Llewellyn
Samuel S. Mackeown
Edward L. Nelson
Harry F. Olson
Greenleaf W. Pickard
Ralph A. Powers
Haraden Pratt
Conan A. Priest
Lynne C. Smeby
Browder J. Thompson
Harold A. Wheeler
Laurens E. Whittemore
Gerald W. Willard
William Wilson
Charles J. Young
Paul D. Zottu

•

Helen M. Stote, *Associate Editor*
William C. Copp, *Advertising Manager*
William B. Cowilich, *Assistant Secretary*

Proceedings of the I·R·E

Published Monthly by
The Institute of Radio Engineers, Inc.

VOLUME 32 — *January, 1944* — NUMBER 1

The Importance of Radio in War ..... H. M. Turner 1
H. M. Turner 2
The Transmission Type of Electron Microscope and Its Optics ..... L. Marton and R. G. E. Hutter 3
Ultimate Bandwidths in High-Gain Multistage Video Amplifiers ..... W. R. MacLean 12
Equivalent T and Pi Sections for the Quarter-Wavelength Line ..... C. G. Brennecke 15
The Receiving Antenna ..... Ronold King and Charles W. Harrison, Jr. 18
The Receiving Antenna in a Plane-Polarized Field of Arbitrary Orientation ..... Charles W. Harrison, Jr. and Ronold King 35
Section Meetings 50
Institute News and Radio Notes
Winter Technical Meeting—1944 51
Board of Directors 53
Executive Committee 53
Correspondence on "A Stabilized Frequency-Modulation System," by Roger J. Pieracci ..... D. L. Jaffe 54
Rochester Fall Meeting—1943 54
Books:
"Practical Radio Communication," by Arthur R. Nilson and J. L. Hornung ..... L. M. Clement 54
"Reference Data for Radio Engineers," Compiled and Published by The Federal Telephone and Radio Corporation ..... Ralph R. Batcher 55
"Physik und Technik der Ultrakurzen Wellen," by H. E. Hollmann ..... W. D. Hershberger 55
"Communication Circuits," by Lawrence A. Ware and Henry R. Reed ..... William Wilson 56
"Electronic Physics," by L. Grant Hector, Herbert S. Lein, and Clifford E. Scouten ..... W. H. Pickering 56
"Mathematics Essential to Electricity and Radio," by Nelson M. Cooke and Joseph G. Orleans ..... Frederick W. Grover 56
Contributors 58
Section Meetings 34A
Membership 34A
I.R.E. People 36A
Positions Open 49A
Advertisers Index 68A

Responsibility for the contents of papers published in the PROCEEDINGS rests upon the authors. Statements made in papers are not binding on the Institute or its members.

Entered as second-class matter October 26, 1927, at the post office at Menasha, Wisconsin, under the Act of February 28, 1925, embodied in Paragraph 4, Section 538 of the Postal Laws and Regulations. Publication office, 450 Ahnaip Street, Menasha, Wisconsin. Editorial and advertising offices, 330 West 42nd St., New York 18, N. Y. Subscription $10.00 per year; foreign, $11.00.

Copyright, 1944, by The Institute of Radio Engineers, Inc.

G-E Antennas . . .

→

G-E TELEVISION RELAY ANTENNA. This relay type of television antenna, developed exclusively by G.E., is in use at General Electric's television "workshop" station WRGB at Schenectady. It has had a remarkable record of performance and reliability since its installation.

This antenna is completely enclosed and contains four horizontal bays. It is highly directional and is especially designed to permit the wide band operation which is so necessary to successful television transmitting. This G-E antenna is so efficient that no relay link should be built without it!

↑ **G-E FM CIRCULAR ANTENNA.** Measurements to date on this horizontally polarized circular antenna show such decisive electrical and mechanical advantages that it has clearly outmoded the conventional types.

Simple, rugged, compact, and pleasing in appearance, the design of this antenna makes it easy to mount on a pole of any diameter. It is grounded to the pole for lightning protection . . . easily adapted for sleet-melting . . . *and easy to tune.* Its wide frequency range and its lower coupling between bays are two of its strongest features. The latter permits optimum power gain per bay, compared to existing designs as evidenced by these figures:

	One-bay	Two-bay	Four-bay
POWER GAIN	.602	1.66	3.47

circular antennas are being operated
surpassing success in six of
ion's important
ations.

The Importance of Radio in War

By Hubert M. Turner

President of the Institute of Radio Engineers

The importance of radio in the present war can scarcely be overestimated. Directly and indirectly it influences many phases of our war operations and is vastly superior to any previously known communication means. Consider the methods used in earlier times. When the Greeks defeated the Persians at Marathon, 490 B.C., the news of victory was carried to Athens, a distance of twenty-two miles, by an Olympian champion runner, who died immediately after he delivered the message. In the War of 1812, the Treaty of Ghent was signed in December, 1814, but hostilities continued for three months before news of the peace, which had to come from Europe by sailing ship, could reach all of the fighting forces. In the meantime the Battle of New Orleans and other important engagements took place with a loss of several thousand lives. In 1860, with war impending, the Pony Express was established between St. Joseph, Missouri, and the Pacific Coast to carry mail in less time than the thirty-four days previously required by stage coach. This cut the regular time to ten days, and the best time was seven days and seventeen hours when Lincoln's inaugural address was carried.

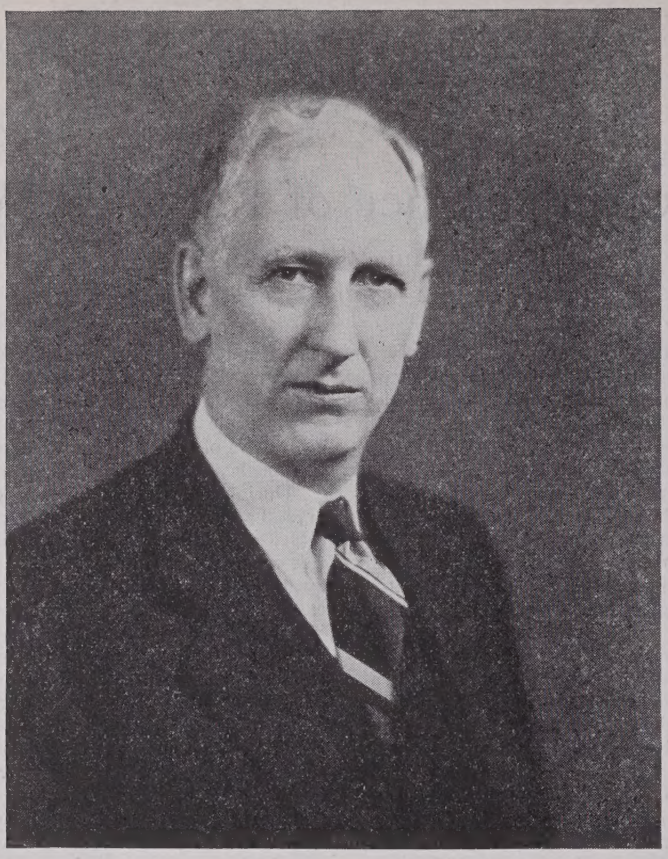
After sixteen months the Pony Express was displaced by a telegraph line which could transmit a message in a fraction of an hour. By the time of the Civil War the telegraph—a much more rapid method of communication between large centers of population where the news was usually distributed through the papers—was used in the eastern states to speed communication. Yet, in an attempt to get news quickly to the people, the editor of the New York Sun at the time of Lincoln's assassination placed a black-bordered flag on a flagstaff in front of his home. The telephone provided a new means of communication for the Spanish War. It was widely used in World War I and continues to play an important part in directing the war effort.

When the United States entered the first World War, radio was available on a limited scale. This was the first war in which it was possible to communicate with ships at sea. In the early part of the war the range of the transmitting stations was limited, and when transmitting to Europe or to ships far from America it was often necessary to have nearer ships relay the message. Fortunately, by September, 1918, the Alexanderson alternator and multiple-tuned antenna were available and provided reliable service with Europe without the necessity of relaying messages. To be able to communicate by radio with the Army Headquarters in France and with London was of very great value because the ocean cables were frequently cut by the enemy and were out of service for considerable periods of time. The ability to communicate with battle areas and with ships, no matter where located, was of tremendous importance, but due to the small number of high-power transmitting stations, they had to be used almost exclusively for official business rather than to keep the public informed of the progress of the war. There was no broadcasting at that time. A limited amount of news received from Europe by radio was transmitted by telegraph or by radio to centers of population and distributed through papers to the public. Often considerable time elapsed before the public received the news; even news of the armistice was not received in remote communities for days and in some cases for weeks.

During the war electron-tube receivers largely replaced the old crystal and other types of receivers. Transmitting and receiving sets were developed for use on airplanes and, so far as I recall, this was the first time that communication with planes had been established. In the early days of the war we were seriously handicapped by lack of radio equipment and facilities for manufacturing it, as well as by lack of trained personnel. However, as a result of the experience gained and the interest created, radio received a tremendous impetus. During the next few years broadcasting was developed and has since been improved until a world-wide broadcasting service was available before we entered World War II. It is now possible in the course of half an hour in the evening to receive first hand reports direct from such military and political centers as: Chungking, London, Ankara, Cairo, Moscow, Australia, the South Sea Islands and many other places. What a source of satisfaction this is to the parents whose sons and daughters are serving their country in distant lands. With sixty-million receivers in the country news reaches practically everyone in the course of a few hours.

Techniques developed in connection with radio are being applied to many other uses. Airport control using ultra-high frequencies is greatly superior to former systems and gives an effective range of thirty miles at one thousand feet elevation. The use of carrier current over power lines to turn on and off the outdoor and obstruction lights at military bases saves 85 per cent of the copper that would be required by separate control circuits. High-frequency heating is used for speeding up the production of laminated airplane-propeller blades and at the same time improving the quality. High-frequency heating is also used in the making of plywood and other laminated structures which are finding increasing use in many fields. Electronic controls are used in electric welding and in speeding up tank production. Several methods for rapid precision weighing have been developed, including one designed for the use of the blind.

Besides the more usual ways in which radio serves our Army and Navy it is reported that paratroopers carry miniature transmitting sets, the "walkie-talkie" performs a most useful function, and tanks are provided with communication facilities. Without radio it would be impossible to conduct a global war and direct battleships, cruisers, submarines, supply ships, air fleets, and vast armies in all parts of the world. It is hoped and expected that radio will contribute to continued peace after the war is over.



Underwood and Underwood

Hubert M. Turner

Hubert Michael Turner was born on July 20, 1882, at Hillsboro, Illinois. After being graduated in electrical engineering from the University of Illinois in 1910, he remained as an assistant instructor for two years while taking graduate work in mathematics, physics, and electrical engineering. He received his Master's degree in 1915. From 1912 to 1918 he instructed at the University of Minnesota and organized courses in transient phenomena and radio. During the war he was placed in charge of technical instruction of the Signal Corps unit of enlisted men at the University of Minnesota. In October, 1918, he became assistant professor of radio with the Signal Corps School for Officer Candidates at Yale.

In 1919 he was appointed assistant professor of electrical engineering at Yale, and in 1926, associate professor. His entire time is devoted to the graduate course in communication engineering and he has developed new methods of presenting theory and many special experimental methods as well as improved laboratory technique. He has had practical experience in both power and communication work, and has done advisory work in several branches of the electrical engineering field.

In 1934 he was a delegate from the National Research Council to the U.R.S.I. meeting in London and delivered a paper on high-frequency measurements. Since 1935 he has been in charge of all the Yale graduate work in electrical engineering and has also introduced the Juniors to electrical engineering at Yale University. He was consultant to the city of Boston on two-way police radio systems, to other cities, and to a number of other groups. He served as an expert witness and advisor to the Counsel in the case of the Hartford Electric Light hearing before the Federal Power Commission. He was president of the New Haven Bird Club during 1940–1942 and president of the New Haven Mineral Club for 1942–1944.

He is a member of the American Institute of Electrical Engineers, the International Union of Scientific Radio Telegraphy, the American Association for the Advancement of Science, the Franklin Institute, and Sigma Xi. He has been active in committee work on matters relating to standardization, technical papers, instruments and measurements, and communications.

Professor Turner became an Associate member of the Institute in 1914; Member in 1920; and Fellow in 1937.

The Transmission Type of Electron Microscope and Its Optics*

L. MARTON†, NONMEMBER, I.R.E., AND R. G. E. HUTTER†, NONMEMBER, I.R.E.

Summary—**The optics of the most widely used type of electron microscope—the transmission type—is described. A survey of the basic theory is given with special emphasis on the limitations of the resolving power.**

APPROXIMATELY ten years ago a new instrument was introduced for research in biology, chemistry, and related fields—the electron microscope. Many scientific and popular papers have been published about the applications of this latest tool of science which extends the range of vision far beyond that of the light microscope. Few publications, however, are devoted to the theory of this instrument and to the problems connected with its further improvement.

The purpose of this paper is to present a brief survey of the theory upon which the electron microscope is based with special emphasis on the limitations of its resolving power. This discussion will distinguish between those limitations that are physically impossible to overcome and those that are amenable to diminution.

The majority of electron microscopes in use today are of the transmission type. Electrons leaving a source, usually a hot filament, are accelerated by a high voltage and are formed into a nearly parallel beam by the "condenser lens." This beam falls upon the object which scatters the electrons to an extent depending on the mass thickness of various sections. Only the electrons scattered within the solid angle formed by the objective aperture will contribute to the image formation. The number of electrons per unit area of a plane behind the object is hence a measure of the distribution of mass in the object. The electrons then pass the "objective lens" which forms an image of the object. This image is then magnified by the "projection lens" in order to make the detail, which must be already contained in the image formed by the objective lens, visible to the human eye. The final image is produced on a fluorescent screen or on a photographic plate. (See Fig. 1.)

The "lenses" mentioned above may be either electrostatic or electromagnetic fields of rotational symmetry about the optical axis. The focusing action of such fields on electron beams is, as we shall see later, similar to that of glass lenses on light beams.

The refracting power of these lenses can be changed by varying the potential for an electrostatic lens or the magnetic field strength for a magnetic lens.

Other types of electron microscopes (scanning type, emission type) are at present of less importance and will be omitted in this discussion.

* Decimal classification: 621.375.1. Original manuscript received by the Institute, April 12, 1943.

† Division of Electron Optics, Stanford University, California.

I. Resolving Power

The resolving power of any microscope is a measure of its ability to distinguish fine detail of the object. It is defined as the smallest distance between two points in the object structure which appear as two separate points in the image produced by the object lens.‡ If the image

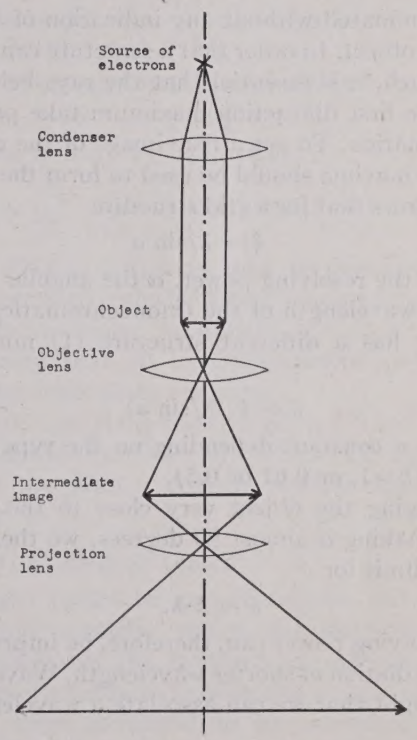


Fig. 1—Optical system of a compound microscope.

of two separate points is a blurred spot, even the greatest magnification in the following stages of the microscope cannot resolve the spot into two discrete points in any projected image.

In light microscopy the limit of the resolving power is of the order of the wavelength of the light used for illumination of the object. The best microscope will show two separate points of an object as distinct points of the image only if their distance is greater than or approximately equal to one half the wavelength of the light. This limitation is one which no microscope can overcome and is due to diffraction of the light waves in the object. (See Fig. 2.)

The first diffraction maximum consists of those light

‡ This definition of the resolving power is not quite correct. It would be better to call the smallest resolved distance "the limit of resolution" and its reciprocal the "resolving power." However, we conform to accepted usage in many publications by calling the distance itself the "resolving power."

rays whose path difference equals the wavelength. Abbe has shown that if the objective lens receives only rays belonging to the central beam, the image plane is

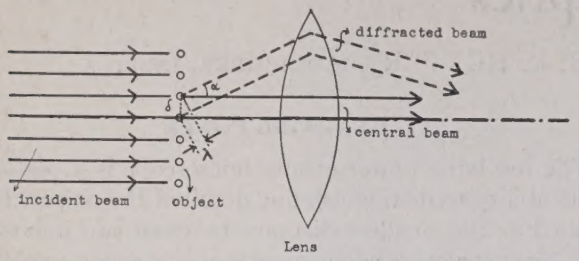


Fig. 2—Diffraction of light rays on a plane grating.

evenly illuminated without any indication of the structure of the object. In order that a structure can be recognized as such, it is essential that the rays belonging at least to the first diffraction maximum take part in the image formation. To get a true image of the object, all diffraction maxima should be used to form the image.

Fig. 2 shows that for a grid structure

$$\delta = \lambda/\sin \alpha \qquad (1)$$

where δ is the resolving power, α the angular aperture, and λ the wavelength of the (monochromatic) light. If the object has a different structure (1) must be replaced by

$$\delta = k \cdot (\lambda/\sin \alpha) \qquad (2)$$

where k is a constant depending on the type of object structure ($k=1$, or 0.61 or 0.5).

By bringing the object very close to the objective lens and making α almost 90 degrees, we then have as the lower limit for

$$\delta \approx k \cdot \lambda. \qquad (3)$$

The resolving power can, therefore, be improved only by using radiation of shorter wavelength. Wave mechanics has taught that we can associate a wavelength with

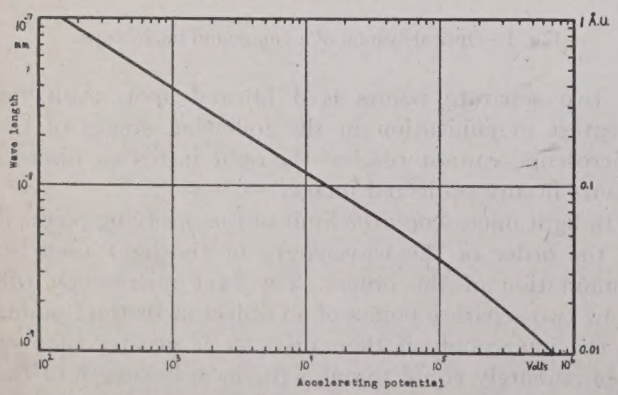


Fig. 3—Wavelength of electrons as a function of the speed expressed in volts.

a moving electron. This wavelength, known as the de Broglie wavelength of an electron, is given by

$$\lambda = \frac{h}{mv} = \frac{12.2}{\sqrt{V}} \cdot \frac{1}{\sqrt{1 + 9.8 \times 10^{-7} V}} \qquad (4)$$

[angstrom units]

where h is the Planck constant, m the mass of the electron, v the velocity, and V the accelerating voltage. This function is represented by the following graph of Fig. 3.

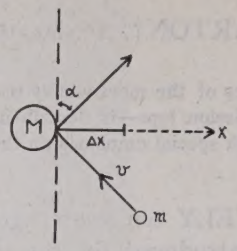


Fig. 4—Resolving-power calculation.

Microscopes using electron waves should, therefore, be far superior to any light microscope as far as resolving power is concerned, if we can assume that it is permissible to substitute the deBroglie wavelength in the Abbe formula (equation (1)). That this assumption is correct was shown by Henneberg[1] by optical considerations and by Henriot[2] who, by starting from the principle of indetermination, came to the same conclusion. (See Fig. 4.)

If an electron with the mass m, velocity v, is moving toward the axis of the microscope at an angle α, striking an object (e.g., a molecule) of mass M, the transversal indetermination (resolving power) is Δx, the indetermination of the momentum being Δp_x.

$$\Delta x \cdot \Delta p_x \approx h = \text{Planck's constant.}$$

The momentum transmitted to M, by the electron encounter is $\Delta p_x = 2mv \cdot \sin \alpha$.

Hence, $\Delta x \cdot \Delta p_x = 2mv \sin \alpha \Delta x \approx h$

$$\Delta x \approx (h/2mv \sin \alpha)$$

the deBroglie wavelength being $\lambda = h/m \cdot v$.

Hence $$\delta = \Delta x \approx (\lambda/2 \sin \alpha). \qquad (5)$$

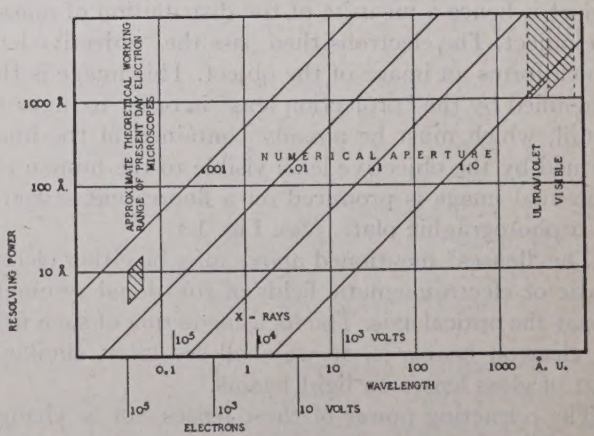


Fig. 5—Resolving power as a function of wavelength for different numerical apertures.

Assuming the values used in light optics for α, the limits of resolving power would be of the order of atomic

[1] Walter Henneberg, "Ueber das Auflösungsvermögen des Elektronenmikroskops für durchstrahlte Objekte," *Zeit. für Instrumentenkunde*, vol. 55, pp. 300–305; 1935.

[2] L. Marton, M. C. Banca, and J. F. Bender, "A new electron microscope," *RCA Rev.*, vol. 5, pp. 232–244; October, 1940.

dimensions. It is at present, however, not possible to form images of atoms and the reason for this lies in the false assumption of the value of α. Other limitations, not yet mentioned, require the use of much lower angular apertures than those used in light optics. These limitations are due to lens aberrations and to the peculiar mechanism of image formation in the electron microscope. Actual values for resolving power of light and electron microscopes are given in Fig. 5.

We shall first consider the imperfections of the lenses resulting from aberrations of a kind similar to those of simple glass lenses. Light optics has, of course, perfected lens systems to such a degree that nowadays they do not have any noticeable aberrations.

II. The Theory of Combined Electrostatic and Electromagnetic Lenses

A. The Field Parameter

In electron microscopes the fields constituting the lenses are symmetrical about the optical axis. The electrostatic potential has to satisfy the Laplace equation, which for space-charge-free fields of the mentioned type, expressed in cylindrical co-ordinates (r, z, θ), is

$$\Delta\phi = \frac{\partial^2\phi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial\phi}{\partial r}\right) = 0. \tag{6}$$

The solution of this equation in a general form is

$$\phi(z, r) = \frac{1}{2\pi}\int_0^{2\pi} \Phi(z + ir\sin\alpha)d\alpha \tag{7}$$

where Φ is an arbitrary real function of the complex quantity $(z+ir\sin\alpha)$. If we expand the integrand of (7) in a Taylor series, we get

$$\Phi(z + ir\sin\alpha) = \Phi(z) + \Phi'(z)\cdot ir\sin\alpha + (1/2!)\Phi''(z)(ir\sin\alpha)^2 + \cdots. \tag{8}$$

Integration with respect to α between the limits $0-2\pi$ gives zero for all odd terms and we have

$$\phi(z, r) = \sum_{\lambda=0}^{\infty} \frac{(-1)^\lambda}{(\lambda!)^2}\Phi^{(2\lambda)}(z)\left(\frac{r}{2}\right)^{2\lambda}. \tag{9}$$

Equation (9) means that the potential $\phi(z, r)$ is known at any point (z, r) in space, if we know the potential and its derivatives along the axis of symmetry.

For a current-free region of a magnetic field

$$\text{curl } H = 0. \tag{10}$$

Introducing the vector potential A, given by

$$H = \text{curl } A \tag{11}$$

and taking into account the symmetry of the field, we have the components of A in cylindrical co-ordinates

$$A_r = 0;\quad A_z = 0;\quad \partial A_\theta/\partial\theta = 0;\quad (A_\theta = A(z, r)). \tag{12}$$

Hence from (11)

$$H_r = -\frac{1}{r}\frac{\partial}{\partial z}(rA_\theta);\quad H_\theta = 0;\quad H_z = \frac{1}{r}\frac{\partial}{\partial z}(rA_\theta). \tag{13}$$

Then (10) means

$$\frac{\partial H_\theta}{\partial\theta} = 0;\quad \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = 0;\quad \frac{\partial H_r}{\partial\theta} = 0. \tag{14}$$

The second equation of (14) can also be written as

$$-\frac{\partial^2}{\partial z^2}(rA_\theta) - \frac{\partial}{\partial r}\left(\frac{\partial A_\theta}{\partial r} + \frac{1}{r}A_\theta\right) = 0$$

or

$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{A}{r}\right) = 0. \tag{15}$$

This last equation is the analogue of the Laplace equation for the electrostatic potential $\phi(z, r)$. Its solution in a general form is given as

$$A = \frac{i}{2\pi}\int_0^{2\pi} \psi(z + ir\sin\alpha)\sin\alpha d\alpha \tag{16}$$

or after expansion

$$A = -\sum_{\lambda=0}^{\infty} \frac{(-1)^\lambda}{\lambda!(\lambda+1)!}\psi^{(2\lambda+1)}(z)\left(\frac{r}{2}\right)^{2\lambda+1} \tag{17}$$

where ψ is the magnetic potential, defined by

$$H = -\text{ grad }\psi = -\psi'. \tag{18}$$

Substituting H for ψ in (17), we get

$$A = \sum_{\lambda=0}^{\infty} \frac{(-1)^\lambda}{\lambda!(\lambda+1)!}H^{(2\lambda)}(z)\cdot\left(\frac{r}{2}\right)^{2\lambda+1}. \tag{19}$$

Equations (9) and (19) show that the field at any point (z, r) is determined if the field functions and their derivatives are known along the optical axis. It is, therefore, not surprising that all lens parameters are given in terms of the fields along the axis.

It is worth remarking that the series (9) and (19) converge in a circle of radius $r=r_0$, where r_0 is equal to the distance of the point z, 0 to the nearest singular point of the field.

B. Geometrical Electron Optics

As in light optics we can distinguish between geometrical and physical optics. Geometrical electron optics investigates the influence of electrostatic and electromagnetic fields on moving electrons in dimensions which are large compared to the wavelength of the electrons. Focusing action of fields, relation between object and image location, magnifications, lens aberration, etc., can be treated without taking into account the wave nature of electrons. An account of wave nature becomes necessary, however, if we want to explain diffraction patterns, either in light optics or as observed in the electron microscope.

Two methods have been used in geometrical electron optics to study the properties of electron lenses, both yielding the same results. The first to calculate the defects of electron lenses, Glaser,[3–6] used the method of geometrical light optics. About one hundred years ago Hamilton developed the analogy between mechanical

[3] Walter Glaser, "Ueber geometrisch-optische Abbildung durch Elektronenstrahlen," *Zeit. für Phys.*, vol. 80, pp. 451–464; February, 1933.

[4] Walter Glaser, "Ueber optische Abbildung durch mechanische Systeme und die Optik allgemeiner Medien," *Ann. der Phys.*, vol. 18, pp. 557–585; November, 1933.

[5] Walter Glaser, "Theorie des Elektronenmikroskops," *Zeit. für Phys.*, vol. 83, pp. 104–122; June, 1933.

[6] Walter Glaser, "Zur Bildfehlertheorie des Elektronenmikroskops," *Zeit. für Phys.*, vol. 97, pp. 177–201; October, 1935.

and optical problems. The starting point of Hamilton's theory of geometrical light optics is Fermat's principle of shortest light time. Glaser extends the theory to continuously refracting media and defines a refracting index of the electromagnetic field. Lens aberrations are treated by making use of a function called "Brun's 'Eikonal.'"

These methods are however, less familiar to electronics engineers. We shall, therefore, briefly describe the less elegant method, used first by Brüche and Scherzer,[7,8] called the "path method." This theory starts out with Lorentz equations, or their equivalent, as follows:

$$\left.\begin{aligned} m\ddot{x} &= -e(x/r)E_r - e\dot{y}H_z + e(y/r)\dot{z}H_r \\ m\ddot{y} &= -e(y/r)E_r + e\dot{x}H_z - e(x/r)\dot{z}H_r \end{aligned}\right\} \text{acceleration equations} \qquad (20)$$

$m/2(\dot{z}^2+\dot{x}^2+\dot{y}^2) = e\cdot\phi$ energy equation

where x and y are directions perpendicular to each other and to the z axis, and the superscript dot ($\dot{}$) means differentiation with respect to time. Furthermore, $r=\sqrt{x^2+y^2}$.

Introducing $w = x + iy$ and $\bar{w} = x - iy$ (21)

and replacing the derivatives with respect to time t by total derivatives with respect to z, $(d/dt = \dot{z}\, d/dz)$, we get a differential equation for the path of an electron through the field

$$\sqrt{\frac{2\phi}{1+w'\bar{w}'}}\,\frac{d}{dz}\left(w'\sqrt{\frac{2\phi}{1+w'\bar{w}'}}\right) = -\frac{w}{r}E_r + i\sqrt{\frac{e}{m}}\cdot\sqrt{\frac{2\phi}{1+w'\bar{w}'}}\left(w'H_z - \frac{w}{r}H_r\right) \qquad (22)$$

where

$$E_r = -\partial\phi/\partial r; \quad H_r = \operatorname{curl}_r A = -\partial A/\partial z;$$
$$H_z = \operatorname{curl}_z A = (1/r)[\partial(rA)/\partial r] \qquad (23)$$

the superscript prime ($'$) means differentiation with respect to z, and $\phi(z, r)$; $A(z, r)$ are given by (9) and (19).

The solution of (22), however, is possible only in unimportant cases. In a general case an approximate method must be used. If electron paths running close to the optical axis are considered, one can treat r, w, $\bar{w}$, w', $\bar{w}'$, as small quantities of the same order of magnitude. Substituting into (21) only the lowest powers of the quantities of the series (9) and (19), a path equation is obtained which is valid for paths near the axis only, so-called paraxial-ray paths.

$$\sqrt{\Phi}\,\frac{d}{dz}(w'\sqrt{\Phi}) = -\frac{w}{4}\Phi'' + i\sqrt{\frac{e\Phi}{8m}}(zw'H + wH'). \qquad (24)$$

The factor i, in one term, shows that due to the magnetic field, there will be a rotation of the electrons around the optical axis, known as Larmor precession. If a system of co-ordinates rotating with the path is introduced a real differential equation results. Let ω_L be the Larmor frequency, then

$$u = w\cdot e^{-i\int\omega_L dt};\quad \omega_L dt = \frac{eH}{2m}dt = \frac{eH}{2m}\cdot\frac{dz}{\dot{z}} = \sqrt{\frac{e}{8m\Phi}}H dz. \qquad (25)$$

Hence $u = w\cdot e^{-i\kappa}$; $\bar{u} = \bar{w}e^{i\kappa}$

$$\kappa = \sqrt{\frac{e}{8m}}\int_{z_0}^{z}\frac{H}{\sqrt{\Phi}}\,dz. \qquad (26)$$

Substitution into (24) gives

$$\Phi u'' + \tfrac{1}{2}\Phi' u' + \tfrac{1}{4}\Phi'' u + (eH^2/8m)u = 0. \qquad (27)$$

This equation describes the "ideal optics," sometimes called "Gauss' Electron Dioptrics."

In the case of pure electrostatic fields $A=0$; in the case of pure magnetic field $\Phi' = \Phi'' = 0$, $(\Phi \neq 0$, since Φ is the accelerating voltage). Equation (27) is a linear differential equation with two linearly independent solutions, forming the so-called fundamental system. A general solution of (27) in terms of elementary functions can only be given for a few types of functions Φ and H. The general solution for one important type of magnetic field will be given later.

Lens aberrations that are deviations from the "ideal" path, as described by (27), can only be computed by starting with a solution of (27). The path is called "ideal" because paths starting at various angles, from a point P in the object space, pass again through a point P' in the image space. P' is the image of P.

C. Lens Aberrations

1. *Aperture Defects.* Taking into account terms in (9) and (19) which are of third-order small, we get a differential equation for electron paths valid for paths farther away from the axis than paraxial-ray paths. Instead of solving this equation a solution of the paraxial-ray equation is assumed to be known and an equation which describes the deviations from the "ideal" path due to the third-order terms is derived. These deviations are called "third-order aberrations." The calculations are long and tedious; therefore, only the final result is stated. Analogous to the procedure in light optics, the deviation $\Delta u_i(\Delta x_i, \Delta y_i)$, in the image plane, is resolved into a number of aberrations

$$(-1)^{n+1}\Delta u_i = \alpha u_0^2\bar{u}_A + \beta u_0\bar{u}_0 u_A + \gamma u_0^2\bar{u}_A + \delta\bar{u}_0 u_A^2 + \epsilon u_0 u_A\bar{u}_A + \zeta u_A^2\bar{u}_A \qquad (28)$$

where u_0 and u_A are the values of the paraxial-ray path at the object and aperture planes, respectively. Each, if present alone, causes a characteristic kind of distortion of the image. It can be seen that all aberrations, except the last one, will disappear if $u_0=0$; that is, if the object point lies on the axis.

The coefficients α to ζ are given by the following definite integrals with respect to z, between the limits z_0, the object co-ordinate and z_i, the image co-ordinate:[9]

[7] E. Brüche and O. Scherzer, "Geometrische Elektronenoptik," Julius Springer, Berlin, Germany, 1934, pp. 115 and 128.

[8] H. Busch and E. Brüche, "Beiträge zur Elektronenoptik," G. A. Barth, Leipzig, Germany, 1937, pp. 24–41.

[9] This useful table of the aberration coefficients appeared first in H. Busch and E. Brüche, "Beiträge zur Elektronenoptik," J. A. Barth, Leipzig, Germany, 1937. Due to war conditions the book is not easily available and, therefore, this table is worth reproducing.

$$\alpha = \frac{M}{16\sqrt{\Phi_0}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\frac{5}{4}\Phi''^2 + \frac{5}{24}\frac{\Phi'^4}{\Phi^2} + \frac{7}{2}\frac{\Phi'^3}{\Phi}\left(\frac{r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{3r_\alpha}\right) - \frac{3}{4}\Phi'^2\left(\frac{r_\gamma'^2}{r_\gamma^2}+\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}\right) + \frac{e}{m}\Phi H'^2 + \frac{3e^2}{8m^2}H^4 - \frac{e\Phi}{2m}H^2\left(\frac{r_\gamma'^2}{r_\gamma^2}+\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}\right) + \frac{35e}{16m}\frac{\Phi'^2}{\Phi}H^2 - \frac{3e}{m}\Phi'HH' - \frac{e\Phi'}{2m}H^2\left(\frac{3r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right)\right] r_\gamma^3 r_\alpha dz + \frac{M}{4}\left[\left(\frac{\Phi''}{8\Phi}+\frac{5\Phi'^2}{32\Phi^2}+\frac{\Phi' r_\gamma'}{2\Phi r_\gamma}+\frac{3}{2}\frac{r_\gamma'^2}{r_\gamma^2}+\frac{eH^2}{8m\Phi}\right) r_\gamma^2\right]_{z_0}^{z_i} + i\frac{M}{16}\sqrt{\frac{2e}{m}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\Phi r_\gamma'^2 - \frac{\Phi'}{2}r_\gamma r_\gamma' + r_\gamma^2\left(\frac{3eH^2}{8m}+\frac{9\Phi'^2}{8\Phi}-\Phi'\frac{H'}{H}\right)\right] H dz + i\frac{M}{32}\sqrt{\frac{2e}{m}}\left[\Phi^{-1/2}\left(\frac{3}{2}\frac{\Phi'}{\Phi}+2\frac{r_\gamma'}{r_\gamma}-\frac{H'}{H}\right) H r_\gamma^2\right]_{z_0}^{z_i}. \tag{29a}$$

$$\beta = \frac{M}{8r_{\alpha A}\sqrt{\Phi_0}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\frac{5}{4}\Phi''^2 + \frac{5}{24}\frac{\Phi'^4}{\Phi^2} + \frac{7}{3}\frac{\Phi'^3}{\Phi}\left(\frac{r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right) + \Phi'^2\left(\frac{r_\gamma'^2}{4r_\gamma^2}+\frac{r_\alpha'^2}{4r_\alpha^2}-2\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}\right) + \frac{e}{m}\Phi H'^2 + \frac{3e^2}{8m^2}H^4 + \frac{e}{m}\Phi H^2\left(\frac{r_\gamma'^2}{2r_\gamma^2}+\frac{r_\alpha'^2}{2r_\alpha^2}-2\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}\right) + \frac{35e}{16m}\frac{\Phi'^2}{\Phi}H^2 - \frac{3e}{m}\Phi'HH' - \frac{e}{m}\Phi' H^2\left(\frac{r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right)\right] r_\gamma^2 r_\alpha^2 dz + \frac{M}{8r_{\alpha A}}\left[\sqrt{\Phi_0}\,\Phi^{-3/2}\Phi' + 4r_\gamma' r_\alpha'\right]_{z_0}^{z_i}. \tag{29b}$$

$$\gamma = \frac{M}{16r_{\alpha A}\sqrt{\Phi_0}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\frac{5}{4}\Phi''^2 + \frac{5}{24}\frac{\Phi'^4}{\Phi^2} + \frac{7}{3}\frac{\Phi'^3}{\Phi}\left(\frac{r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right) - \Phi'^2\left(\frac{5}{4}\frac{r_\gamma'^2}{r_\gamma^2}+\frac{5}{4}\frac{r_\alpha'^2}{r_\alpha^2}-\frac{r_\alpha'r_\gamma'}{r_\alpha r_\gamma}\right) + \frac{e}{m}\Phi H'^2 + \frac{3e^2}{8m^2}H^4 - \frac{e}{m}\Phi H^2\left(\frac{3r_\gamma'^2}{2r_\gamma^2}+\frac{3r_\alpha'^2}{2r_\alpha^2}-2\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}\right) + \frac{35e}{16m}\frac{\Phi'^2}{\Phi}H^2 - \frac{3e}{m}\Phi'HH' - \frac{e}{m}\Phi' H^2\left(\frac{r_\gamma'}{r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right)\right] r_\gamma^2 r_\alpha^2 dz + \frac{M}{4r_{\alpha A}}\left[r_\gamma' r_\alpha'\right]_{z_0}^{z_i} + i\frac{M}{8r_{\alpha A}}\sqrt{\frac{2e}{m}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\Phi r_\gamma' r_\alpha' - \frac{\Phi'}{4}(r_\gamma r_\alpha' + r_\alpha r_\gamma') + r_\gamma r_\alpha\left(\frac{3eH^2}{8m}+\frac{9\Phi'^2}{8\Phi}-\Phi'\frac{H'}{H}\right)\right] H dz + i\frac{M\sqrt{\Phi_0}}{16r_{\alpha A}}\sqrt{\frac{2e}{m}}\left[\frac{H}{\Phi}\right]_{z_0}^{z_i}. \tag{29c}$$

$$\epsilon = 2\bar{\delta} = \frac{M}{8r_{\alpha A}^2\sqrt{\Phi_0}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\frac{5}{4}\Phi''^2 + \frac{5}{24}\frac{\Phi'^4}{\Phi^2} + \frac{7}{2}\frac{\Phi'^3}{\Phi}\left(\frac{r_\gamma'}{3r_\gamma}+\frac{r_\alpha'}{r_\alpha}\right) - \frac{3}{4}\Phi'^2\left(\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}+\frac{r_\alpha'^2}{r_\alpha^2}\right) + \frac{e}{m}\Phi H'^2 + \frac{3e^2}{8m^2}H^4 - \frac{e\Phi}{2m}H^2\left(\frac{r_\gamma'r_\alpha'}{r_\gamma r_\alpha}+\frac{r_\alpha'^2}{r_\alpha^2}\right) + \frac{35e}{16m}\frac{\Phi'^2}{\Phi}H^2 - \frac{3e}{m}\Phi'HH' - \frac{e\Phi'}{2m}H^2\left(\frac{r_\gamma'}{r_\gamma}+\frac{3r_\alpha'}{r_\alpha}\right)\right] r_\gamma r_\alpha^3 dz + \frac{M}{4r_{\alpha A}^2}\left[r_\alpha'^2\right]_{z_0}^{z_i} + i\frac{M}{8r_{\alpha A}^2}\sqrt{\frac{2e}{m}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\Phi r_\alpha'^2 - \frac{\Phi'}{2}r_\alpha r_\alpha' + r_\alpha^2\left(\frac{3eH^2}{8m}+\frac{9\Phi'^2}{8\Phi}-\Phi'\frac{H'}{H}\right)\right] H dz. \tag{29d}$$

$$\zeta = \frac{M}{16r_{\alpha A}^3\sqrt{\Phi_0}}\int_{z_0}^{z_i}\Phi^{-3/2}\left[\frac{5}{4}\Phi''^2 + \frac{5}{24}\frac{\Phi'^2}{\Phi^2} + \frac{14}{3}\frac{\Phi'^3 r_\alpha'}{\Phi r_\alpha} - \frac{3}{2}\Phi'^2\frac{r_\alpha'^2}{r_\alpha^2} + \frac{e}{m}\Phi H'^2 + \frac{3e^2}{8m^2}H^4 - \frac{e}{m}\Phi H^2\frac{r_\alpha'^2}{r_\alpha^2} + \frac{35e}{16m}\frac{\Phi'^2}{\Phi}H^2 - \frac{3e}{m}\Phi'HH' - \frac{2e}{m}\Phi' H^2\frac{r_\alpha'}{r_\alpha}\right] r_\alpha^4 dz. \tag{29e}$$

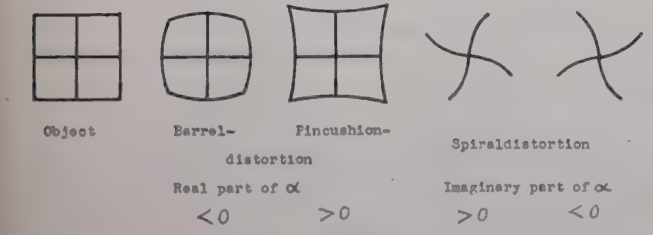


Fig. 6—Distortion.

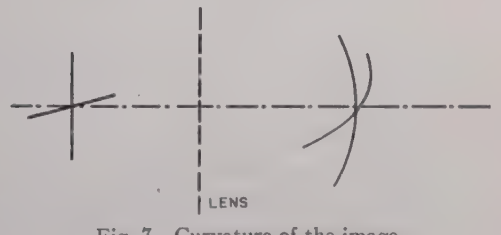


Fig. 7—Curvature of the image.

In the integrand r is a special solution of the paraxial-ray equation (27), Φ is the potential, and H is the magnetic field strength along the axis.

Here $\alpha u_0^2 u_A$ describes the "distortion." The real part of α causes the "barrel" and "pincushion" distortion and the imaginary part the spiral distortion. (Fig. 6.)

$\beta u_0 \bar{u}_0 \bar{u}_A$ describes the fact that the image "plane" is actually a curved surface. (Fig. 7.)

Fig. 8—Astigmatism.

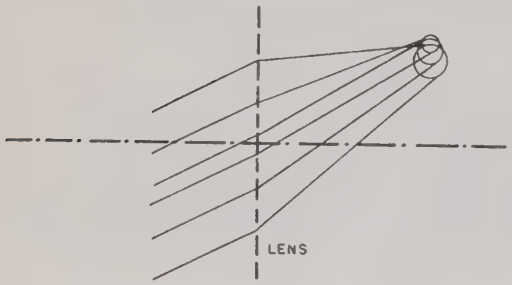


Fig. 9—Coma.

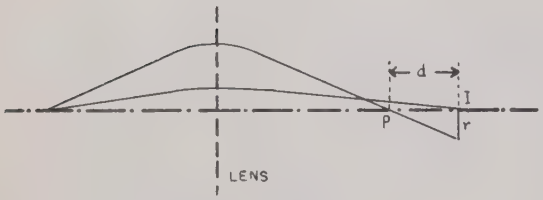


Fig. 10—Spherical aberration.

The radii of the two curves of the image of Fig. 7 are identical. The following term $\gamma u_0^2 \bar{u}_A$, describing "astigmatism," also causes a curvature of the field; the two radii of curvature of the image are, however, not equal. (Fig. 8.)

$\epsilon u_0 u_A \bar{u}_A$ and $\delta \bar{u}_0 u_A^2$ are called "coma" because they describe the fact that an object point gets, at the image plane, a comet appearance. (Fig. 9.)

The defects mentioned so far disappear for object points on the optical axis because $u_0 = 0$ for such points. The last term $\zeta u_A^2 \bar{u}_A$, however, is independent of u_0. Electrons leaving a point on the axis at any angle greater than those of the paraxial beam will not pass through the Gaussian image point I but will intersect the optical axis at a point P between the lens and the ideal image point I. The defect $\zeta u_A^2 \bar{u}_A$ is called spherical aberration and is the most serious of all defects. (Fig. 10.)

The distance $\overline{PI} = d$, or the radius of the circle of confusion at the image point, can be used as a measure for the spherical-aberration defect.

To reduce spherical aberration in electron microscopes the aperture has to be made very small (to limit the values u_A!). Instead of using the quantity u_A in our expression for the physical aperture, we can introduce the angular aperture α and write (29e) as

$$\zeta = \alpha^3 \cdot C \tag{30}$$

where C is essentially the definite integral of (29e). Another way would be to use lenses for which C is zero or at least small. This leads us to state briefly the difficult mathematical problem of determining a field for which the integral in ζ has a minimum value. Scherzer[10] showed by transforming the definite integral into the following equivalent form that it is impossible to make ζ equal zero.

$$\zeta = \alpha^3 \text{ const.} \int_{z_0}^{z_i} \Phi^{1/2} \left[\frac{5}{4} \left(\frac{\Phi''}{\Phi} + \frac{\Phi'}{\Phi} \frac{r_\alpha'}{r_\alpha} - \frac{\Phi'^2}{\Phi^2} \right)^2 + \frac{\Phi'^2}{\Phi^2} \left(\frac{r_\alpha'}{r_\alpha} + \frac{7}{8} \frac{\Phi'}{\Phi} \right)^2 + \frac{e}{m\Phi} \left(H' + H \frac{r_\alpha'}{r_\alpha} - \frac{5}{4} H \frac{\Phi'}{\Phi} \right)^2 + \frac{e}{m} \frac{H^2}{\Phi} \left(\frac{r_\alpha'}{r_\alpha} + \frac{1}{4} \frac{\Phi'}{\Phi} \right)^2 + \frac{1}{64} \frac{\Phi'^4}{\Phi^4} + \frac{eH^4}{4m^2\Phi^2} + \frac{e}{32m} H^2 \frac{\Phi'^2}{\Phi^3} \right] r_\alpha^4 dz \tag{31}$$

where r_α is a special solution of the paraxial-ray equation. ζ can be zero only if $\Phi' = H = 0$, since we have a definite integral over a sum of squares ($\Phi > 0$). Equation (31) results from (29e) by applying partial integration several times noticing that $r_\alpha = 0$ at $z = z_0$ and $z = z_i$.

A problem which could be set up reasonably is the following. Find a field Φ, H so that ζ has a minimum value and the paraxial-ray equation has a solution with two subsequent zeros under the field (i.e., for the same values of z for which Φ and H are defined). Such a problem has been solved only for the case of a symmetrical, weak electrostatic single lens, a case of no practical importance.[11] The existence of a field with a minimum of spherical aberration can be doubted. Rebsch[12] shows that theoretically, at least, the spherical aberration can be reduced below any prescribed limit, by causing the field to vary strongly over a short distance in the neighborhood of the object. Thus for every field there can be made to exist another field of less spherical aberration. The assumption of a field with minimum spherical aberration would, therefore, be wrong. In this connection, it is interesting to see the mistake made by Glaser when he claimed to have found a magnetic field having zero spherical aberration.[13] He changed, by partial integration, the expression for ζ, specialized for the case $\Phi' = \Phi'' = 0$, to

$$\zeta = \text{const. } \alpha^3 \int_{z_0}^{z_i} \left(\frac{2e}{m\Phi} H^4 + 5 \cdot H'^2 - H \cdot H'' \right) r_\alpha^4 dz. \tag{32}$$

The integrant equated to zero gives a differential equation for $H(z)$, which Glaser was able to solve. This field would not have any spherical aberration ($\zeta = 0$). In an answer to Glaser, Rebsch[14] showed that the paraxial-ray

[10] Otto Scherzer, "Ueber einige Fehler von Elektronenlinsen," *Zeit. für Phys.*, vol. 101, pp. 593–603; July, 1936.

[11] Otto Scherzer, "Die schwache elektrische Einzellinse geringster sphärischer Aberration," *Zeit. für Phys.*, vol. 101, pp. 23–26; June, 1936.

[12] R. Rebsch, "Das theoretische Auflösungsvermögen des Elektronenmikroskops," *Ann. der Phys.*, vol. 31, pp. 551–560; March, 1938.

[13] Walter Glaser, "Ueber ein von sphärischer Aberration freies Magnetfeld," *Zeit. für Phys.*, vol. 116, pp. 19–33; 1940.

[14] R. Rebsch, "Ueber den Oeffnungsfehler der Elektronenlinsen," *Zeit. für Phys.*, vol. 116, pp. 729–733; 1940.

path does not change its slope by more than 2 per cent under the field $H(z)$, which was defined only on a segment of the z axis and had an infinite point at one end of the interval. The "ideal" field has, therefore, no refracting power and does not constitute a lens.

2. *Chromatic Aberration.* The curvature of an electron path in an electrostatic or electromagnetic field will vary with the accelerating voltage of the electrons. Two electrons leaving the same point, on the optical axis of a rotationally symmetrical field, with two different speeds will forms two images, of which the one formed by the electron of higher speed will be farther away from the lens. (Fig. 11.)

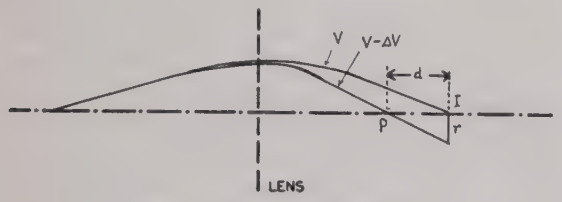


Fig. 11—Chromatic aberration.

As in spherical aberration, the quantities d or r (Fig. 11) can be taken as a measure of chromatic aberration.

Glaser[15] derived an expression for the quantity r or rather for a quantity $\delta_{Cr}=r/M$ where M is the magnification of the lens. If the distribution of velocity in the electron beam lies between V and $V+\Delta V$, assuming $\Delta V/V \ll 1$,

$$\delta_{Cr} = \alpha \frac{\Delta V}{V} \int_{z_0}^{z_i} y' dz = \alpha \cdot \frac{\Delta V}{V} C_{Cr} \qquad (33)$$

where α is the angle of the beam starting at $z=z_0$ and $y=y(z)$ is a special solution of the paraxial-ray equation with $y(z_0)=0$ and $y'(z_0)=1$.

3. *Effects of Aberrations on the Numerical Aperture.* It has been shown that the most serious defects are the spherical and chromatic aberrations since they do not vanish for object points on the axis. From (30) it can be seen that the spherical aberration increases with the third power of the angular aperture while the chromatic aberration is directly proportional to α. Since the constants C_{Sp} and C_{Cr} are large, due to the imperfections of the lenses, α has to be kept small in order to reduce the detrimental effect of these aberrations. While in light optics α is almost $\pi/2$, values for α in electron optics range from 10^{-5} to 10^{-2} radian.

III. Image Formation in Transmission-Type Microscopes

A. Image Formation

The second limiting factor on the resolving power, the peculiar mechanism of image formation, now will be discussed. In a transmission type of electron microscope a nearly parallel beam is directed on the object. The angular aperture α of this beam can be varied by changing the strength of the condenser lens, and is defined as the solid angle subtended at the center of the object by the image of the crossover formed by the condenser lens.[16] (Fig. 12.)

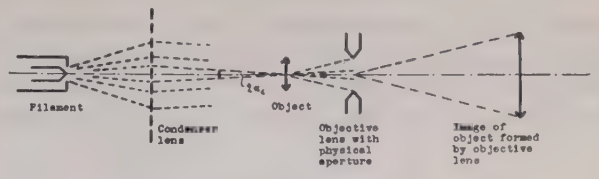


Fig. 12—Condenser, objective-lens system.

The electrons, while passing through the object, are scattered to an extent depending on the structure of the object. To produce an observable contrast in the image, a certain number of electrons have to be scattered beyond the cone defined by the aperture of the objective lens and thus prevented from reaching the observing screen or the photographic plate. The number of electrons scattered within the solid angle of the objective aperture is

$$n = n_0 e^{-xSN} \qquad (34)$$

where n_0 is the number of incident electrons, x the thickness of the specimen, S the total cross section of the atom for scattering outside the angle limited by the objective aperture, and N the number of scattering atoms per cubic centimeter (equal to $N_0 \cdot \rho/M$, with N_0, Avogadro's number, ρ the density, and M the molecular weight). The scattering cross section of the atom has been calculated for a number of practical cases; it is a function of the properties of the atoms composing the specimen (properties such as atomic number and average binding energy of the individual electrons in the atom) and of the speed of the electrons in the beam. It is

$$S = \sigma_e + \sigma_i + \sigma_f \qquad (35)$$

where σ_e, σ_i, σ_f are the cross sections corresponding to elastic, inelastic, and free-electron scattering, respectively, and are given by Marton and Schiff.[17]

The method of observation just described is called "bright-field" observation as opposed to the "dark-field" observation which, as in light ultramicroscopes, uses either sidewise illumination of the object or coaxial illumination with a suitable central stop in the aperture plane of the condenser lens. In principle such observation offers, again as in light optics, the advantage of extending the range of vision. It has been calculated that atoms having atomic numbers higher than seven should be observable.[18] Experimental work in this direction has not progressed very far; indeed, the best dark-field pictures are still behind the quality of the transmission pictures.

In comparison with the light microscope an electron microscope has a much greater "depth of field." This

[15] Walter Glaser, "Die Farbabweichung bei Elektronenlinsen," *Zeit. für Phys.*, vol. 116, pp. 56–67; 1940.

[16] We shall see later that many of the definitions of angular aperture will have to be modified due to the influence that the objective-lens field, extending from the object towards the condenser lens, has on the electron beam passing through the object.

[17] L. Marton and L. I. Schiff, "Determination of object thickness in electron microscopy," *Jour. Appl. Phys.*, vol. 12, pp. 759–765; October, 1941.

[18] L. I. Schiff, "Ultimate resolving power of the electron microscope," *Phys. Rev.*, vol. 61, pp. 721–722; June, 1942.

quantity is defined by the following expression

$$D_{\text{field}} = \Delta z/d = 1/\sin \alpha_0 \qquad (36)$$

where Δz is the distance which the object can be displaced along the axis without causing the blurring of the two object points a distance d apart. α_0 is the effective objective aperture. (Fig. 13.) D_{field} is about five hundred

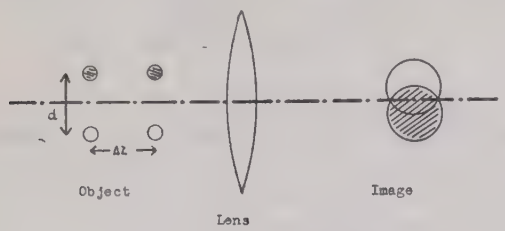


Fig. 13—Depth of field.

times greater than that of light microscopes due to the previously mentioned fact that the angular aperture in electron microscopes is very small compared to that of light microscopes. In light microscopy there are two methods for the investigation of the space structure of an object, stereoscopic observation and "optical sectioning." Due to the greater depth of field of an electron microscope, stereoscopic observation is possible at much higher magnifications than those used in light optics. Optical sectioning is called the method of focusing object planes of different depth perpendicular to the optical axis.[19] The use of this method in electron microscopy is impracticable because all planes within a distance equal to D_{field} are sharply focused and D_{field} is large compared to the microscopic dimension to be studied.

The same reason, the smallness of α_0, explains the great "depth of focus" of an electron lens. This quantity is defined as the distance which the photographic plate can be moved along the axis without causing a blurred image. The angular aperture of the object side α_0 and of the image side α_i are related by the equation

$$\alpha_i = \alpha_0/M, \qquad D_{\text{focus}} = 1/\sin \alpha_i \qquad (37)$$

where M is the magnification (of the order of 100). α_i will be very small; hence the photographic-plate position is not critical at all.[20] (Fig. 14.)

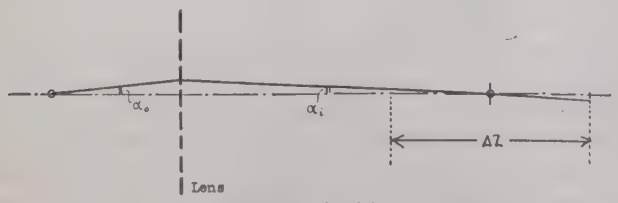


Fig. 14—Depth of focus.

B. Diffraction

Abbe's theory of image formation in light microscopy distinguishes between two images, the primary image or diffraction pattern S_D in the image plane of the light source and the secondary image or the image S_I of the object. (Fig. 15.)

[19] Francis F. Lucas, "The architecture of living cells," *Proc. Nat. Acad. Sci.*, vol. 16, pp. 599–607; September, 1930.

[20] Δz up to 1 meter has been used.

Due to the wave nature of the electrons, both images can also be observed in the electron microscope. If the object is, for instance, a thin metal foil consisting of many crystals with axes oriented in various directions forming a space-grid structure, the well-known electron-diffraction pattern is obtained in the image plane of the cathode, and the image of the metal foil appears in the image plane of the object. Boersch[21] performed a number of experiments showing that the image of the object can be influenced by varying the size of the aperture in the plane of the diffraction pattern. The theory of Abbe that it is essential to utilize the rays of at least the first dif-

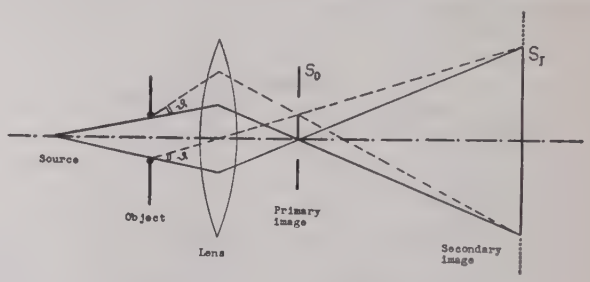


Fig. 15—Diffraction. Primary and secondary image.

fraction maximum in order to obtain true image formation is, therefore, shown to apply also to electron optics.

C. Chromatic Aberration Caused by the Object

Electrons passing through the object in the described manner will not only be scattered and diffracted but will also lose velocity, depending on the properties of the object. It has been shown that the image co-ordinate is a function of the accelerating potential V. One would, therefore, expect a blurring of the image due to any voltage variation ΔV. This variation of voltage, or, what is tantamount to the same thing, velocity, can also be caused by fluctuation of the power sources, and by the initial voltage or velocity distribution of the emitted electrons. The latter factor is small enough to be neglected. The former, however, has to be taken care of by proper voltage regulation of the power sources.

Loss of speed in the object is a factor which cannot be remedied; it is an inherent defect due to the peculiar mechanism of image formation in the electron microscope.

According to measurements carried out by Lenard[22] we have, for the relative loss in velocity of thin objects,

$$(\Delta V)/V = (\epsilon_0/\rho) \cdot \rho \cdot x \qquad (38)$$

where ϵ_0/ρ is a unique function of V, ρ, the density of the object in grams per cubic centimeter, and x, the thickness of the object in millimeters.

Equations (28) and (38) allow the computation of the chromatic aberration due to a loss in velocity of the electrons while passing the object. The result, however, will be larger than the actual measured values since the

[21] H. Boersch, "Ueber das primäre und sekundäre Bild im Elektronenmikroskop," *Ann. der Phys.*, vol. 26, pp. 631–644; June, 1935.

[22] As quoted by W. Bothe, "Handbuch der Physik," 22/2, p. 33; 1933.

function $\epsilon_0/\rho = f(V)$ is based on a mean value for the decrease of velocity as measured on thicker layers than used in the electron microscope.

IV. Limits of the Resolving Power

Thus far the four principal reasons for the limits of the resolving power have been discussed: (1) Spherical aberration due to imperfection of the electron lenses as compared with glass lenses; (2) chromatic aberration so far as it cannot be avoided due to the mechanism of image formation; (3) diffraction defect, as is the limitation for any kind of microscope; and (4) scattering in the case of thick objects.

Besides these defects there are some other causes for reduction in resolving power which are, however, either smaller or can be taken care of by proper design and adjustment of the microscope and its accessories. It has already been mentioned that all other lens aberrations can be reduced to zero by placing the object on the optical axis. Stray magnetic fields deflecting the electron beam can be eliminated by proper shielding. Fluctuations of the power sources can be reduced to negligible proportions by using highly regulated power supplies. Initial voltage distribution of the emitted electron beams and space-charge effects of the electron beam have negligible influence.

The quantities describing the three main defects will be called δ_{Sp} (spherical aberration), δ_{Cr} (chromatic aberration), and δ_D (diffraction). These δ's are distances of two object points whose circles of confusion at the image side overlap to their centers (assuming that always only one defect is present). (Fig. 16.)

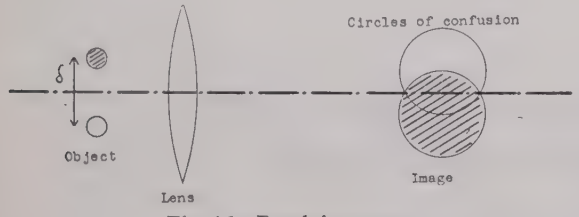


Fig. 16—Resolving power.

All three defects are functions of the effective objective aperture α_0. From (30)

$$\delta_{Sp} = \alpha_0^3 C_{Sp} \tag{39}$$

from (33)

$$\delta_{Cr} = \alpha_0 C_{Cr} \tag{40}$$

and from (1) for small α_0

$$\delta_D = \lambda/\alpha_0. \tag{41}$$

Assuming various values for the constants C_{Sp}, C_{Cr}, and the wavelength λ, the following plot is obtained. (Fig. 17.)

For thin objects the defect due to that chromatic aberration which is caused by the object can be neglected by comparison with that due to spherical aberration. It can be seen that δ_D and δ_{Sp} vary in an opposite manner with respect to α_0. The effective objective aperture α_0 has to be chosen in such a way, therefore, that the two defects become of approximately equal magnitude.

Borries and Ruska[23] calculated the minimum value of δ if spherical aberration and diffraction only are present. In the case of lenses, where the object is within the objective lens, these definitions of angular apertures have to be modified. In the case of thick objects the mechanism of image formation causes a further decrease in the resolving power due to scattering of the electrons in layers between the object and the objective lens. If a thin object were freely suspended in space (Fig. 18A) no further scattering could occur. The usual method of

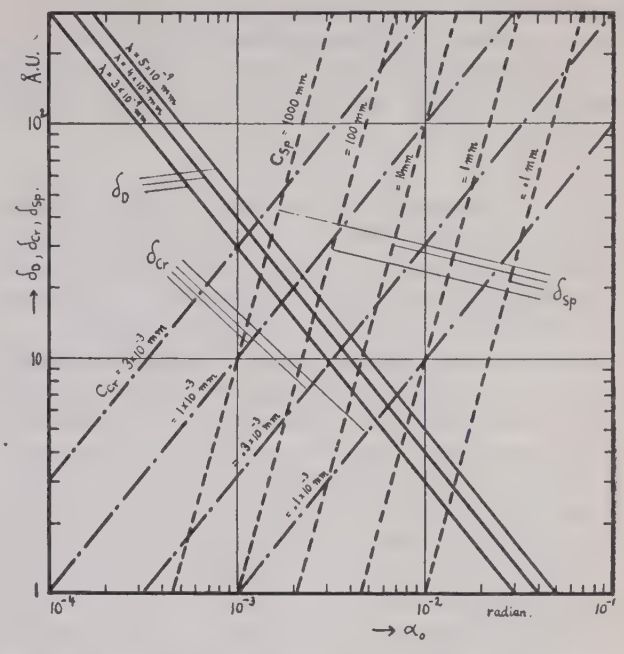


Fig. 17—Diffraction defect, spherical and chromatic aberration in function of the objective aperture.

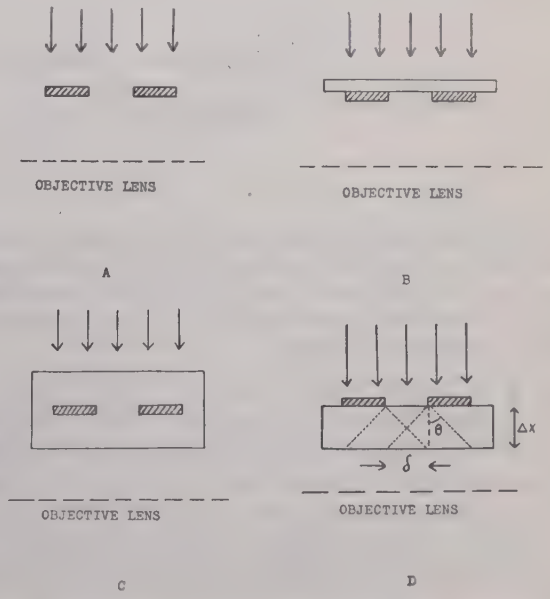


Fig. 18—Different kinds of specimens and the effect of scattering on the resolving power.

[23] B. v. Borries and E. Ruska, "Mikroskopie hoher Auflösung," *Erg. der exakten Naturwiss.*, Julius Springer, Berlin, Germany, 1940, pp. 237–322.

observation, object on objective lens side of the uniform carrier film (Fig. 18B), has the same effect. In case of thick objects (Fig. 18C), where the particle to be resolved is imbedded in the object, the effect is similar to the condition where the particle is placed on that side of a film facing the incoming beam. (Fig. 18D.)

After passing the object the electrons are scattered in the film, thereby causing a reduction of the resolving power. The limit is reached when the distance of two particles becomes

$$\delta = \Delta x \tan \theta. \qquad (42)$$

The scattering angle is a function of the thickness Δx, the accelerating potential V, and constants of the material (density, atomic number, and atomic weight). θ has been calculated on the basis of classical[24] and quantum theory[17] both for single and multiple scattering.

The remarks in the introduction about the superiority of the electron microscope to the light microscope with respect to resolving power might lead one to think that an increase in accelerating potential would increase resolving power. Although the wavelength of the electrons becomes shorter with higher speed, an actual decrease in resolving power will occur. This decrease is still small for voltages between 60 and 200 kilovolts but becomes larger for higher voltages. It is due to the fact that the minimum focal length of a magnetic lens increases proportionally with the accelerating potential. Rebsch[12] showed that the least resolvable distance in an object is proportional to the eighth root of the cube of the accelerating potential. From the point of view of higher resolving power, it is, therefore, not advisable to use higher voltages in electron microscopes. The chromatic aberration, on the other hand, will decrease with higher electron speeds for the same object thickness. Viewing of thicker objects due to higher penetrating power is possible.

Since an actual decrease in resolving power is to be expected, other reasons for applying higher voltages in an electron microscope must exist to justify the slight sacrifice in resolution. Due to higher penetration of the faster electrons and the decreased energy loss in the object, we are able to observe thicker specimens and reduce the heating of the specimen. This last fact makes the observation of living specimens possible. The second point in favor of higher voltages is the reduction of chromatic aberration due to decreased velocity loss in the object.

V. Conclusions

The main factors limiting the resolving power of present-day electron microscopes have been discussed. Some of these, such as those caused by diffraction and the mechanism of image formation, are inherent limitations; others, like aperture defects and, to some extent, chromatic aberrations, can be reduced by developing better lenses.

The mathematical problem of improvement of lenses is difficult but not entirely unpromising. In the second part of this paper, we intend to show how, coupled with experimental methods, serious progress may be expected in spite of such limitations as saturation of the pole pieces of magnetic lenses and cold emission in electrostatic lenses.

After the first ten years of rapid development of electron microscopy, a much slower tempo and more tedious work will characterize the future work in this latest field of electronics.

[24] L. Marton, "Quelques considerations concernant le pouvoir separateur en microscopie electronique," *Physica*, vol. 3, pp. 959–967; November, 1936.

Ultimate Bandwidths in High-Gain Multistage Video Amplifiers*

W. R. MacLean†, Associate, I.R.E.

Summary—**This paper considers the design of high-gain video amplifiers from the point of view of choosing the number of stages. For a plain resistance-coupled and for a critically compensated shunt-peaked amplifier a simple rule is given for determining the number of stages for maximum bandwidth.**

MUCH has been written in recent years on the high-frequency response of video amplifiers, and many improvements on plain resistance-capacitance coupling have been devised. Such for instance, are shunt peaking, series peaking,[1] filter coupling,[2] etc. There has also been considerable difference of opinion on methods of compensation; even where agreement on circuit has been reached, the question of optimum parameters remains.[3]

These differences arise naturally in the absence of a mathematical criterion of merit. Such a criterion is not at hand, and indeed may not exist in any universal form. A goodness factor applied to the operation of one stage of amplification may be inapplicable to a multistage amplifier even in principle. Any optimum design depends on the boundary conditions of the problem: certain

* Decimal classification: R363.1. Original manuscript received by the Institute, April 1, 1943; revised manuscript received, August 12, 1943.

† Polytechnic Institute of Brooklyn, Brooklyn, N. Y.

[1] A discussion of these two methods is given by S. W. Seeley and C. N. Kimball in "Analysis and design of video amplifiers," *RCA Rev.*, vol. 3, pp. 290–309; January, 1939.

[2] H. A. Wheeler, "Wide-band amplifiers for television," Proc. I.R.E., vol. 27, pp. 429–438; July, 1939.

[3] A. Preisman, "Some notes on video-amplifier design," *RCA Rev.*, vol. 2, pp. 421–433; April, 1938. His parameter, k, is the reciprocal square root of the k used here.

characteristics are maximized while others are kept constant. The result depends naturally on the choice of constant parameters. In a high-gain video input amplifier, the constant quantity is the over-all gain required: thereafter one strives for either a desired bandwidth or a maximum bandwidth.

In studying the performance of a video amplifier as a whole, a somewhat different approach is needed from that taken when studying the action of just one stage. For example, suppose by some kind of compensation, one has devised a stage whose gain fluctuates around a constant value out to a high frequency. These deviations may appear small, but if many such stages were combined, they might become excessive by superposition. The entire amplifier could perhaps be considered flat only out to the neighborhood of the first fluctuation. In such a case it might be better to start with single stages of less apparent bandwidth but which are a little flatter at the beginning.

It is often said that gain and bandwidth are exchangeable. That is true from a certain point of view for a single-stage amplifier, but the implication is that the loss in gain may be made up by using more stages. Yet if this is done, each stage must be better. Is this a limitation? Is there an optimum number of stages,[4] or does the bandwidth increase indefinitely with the number of stages? In the following, such a problem is posed and analyzed for two simple cases.

One Resistance-Capacitance Stage

In a single wide-band pentode stage, the *nominal* gain g is merely

$$g = rg_m \qquad \text{presuming, } r \ll r_p \tag{1}$$

where r is the plate-load resistor, and g_m is the transconductance. At high frequencies, the *total* shunt capacitance c comes into play, and r in (1) must be replaced by z, the actual load impedance. The relative gain ρ is defined as the ratio of actual to nominal gain; that is,

$$\rho = z/r. \tag{2}$$

The above can be expressed readily in terms of r, C, and ω, where ω is the radian frequency. In this form, however, it does not have a generalized character. To attain such generality, first a guide frequency ω^0 is defined by

$$\omega^0 rC = 1 \tag{3}$$

and then a frequency parameter ϕ by

$$\phi = \omega/\omega^0. \tag{4}$$

In this terminology, (2) becomes

$$\rho = 1/(1 + j\phi). \tag{5}$$

Use of a guide frequency such as ω^0 is often made in the literature and frequently causes confusion, especially in problems where more than one such guide is involved. It is essentially a yardstick of frequency, but by (3) depends on r, which is merely an incidental parameter. A better formulation is found by expressing ω^0 and hence ϕ and ρ in terms of first-rank parameters. For that purpose one defines, as usual,

$$m = g_m/C \tag{6}$$

as the *figure of merit* of the tubes involved. By the use of m and (1), ω^0 can be given as

$$\omega^0 = m/g \tag{7}$$

whence,

$$\phi = \omega/(m/g). \tag{8}$$

The yardstick m/g is more information than just ω^0.

ρ is the complex relative gain and as such contains information on both phase and amplitude The important question of the relative badness of phase and amplitude distortion will be omitted in this discussion and attention directed to amplitude response alone. This would be given by the quantity $|\rho|$:

$$|\rho|^2 = 1/(1 + \phi^2). \tag{9}$$

What *bandwidth* is determined by (9)? In other words, what is now the highest usable ω, say, $\bar{\omega}$? Often one speaks of ω^0 as this quantity. From (4) and (9) this is the frequency for which $\phi = 1$, i.e., the half-power point. But clearly, for many applications, one cannot afford a drop to the half-power point. In general the quantity $\bar{\omega}$ depends on the tolerance allowed.

Suppose it is required that

$$1 - \delta \leqq |\rho| \leqq 1 + \delta \tag{10}$$

throughout the usable frequency range.

δ is called the *tolerance*. Since from (9), $|\rho|$ is always less than unity, one can find $\bar{\omega}$ by solving

$$1/(1 + (\bar{\omega}g/m)^2) = (1 - \delta)^2$$

for it. If we limit ourselves to precision amplifiers, i.e., to the case $\delta \ll 1$, higher-order terms in δ may be dropped, and one has $1 + (\bar{\omega}(g/m))^2 = 1 + 2\delta$.

$$\bar{\omega}g = m\sqrt{2\delta} \tag{11}$$

as the bandwidth formula for a one-stage resistance-capacitance amplifier.

Resistance-Capacitance Ultimate

In (11) one sees the "exchangeability" of bandwidth and gain in the product on the left. But a difficulty arises if so much gain is exchanged that g is no longer above unity: then no *amplifier* remains.

This observation leads to the following problem: a (multistage) amplifier is to be built with over-all gain G out of a number N of identical stages. The tolerance is fixed at Δ. Can the bandwidth be indefinitely increased by increasing N or is there a maximum bandwidth?

First there is the relation between the over-all tolerance and that per stage, namely,

$$(1 - \delta)^N = 1 - \Delta. \tag{12}$$

If, as before, we limit ourselves to precision amplifiers, then $\Delta \ll 1$, and a fortiori $\delta \ll 1$. Hence, one has merely

$$N\delta = \Delta. \tag{13}$$

[4] The television system used determines whether there must be an odd or an even number. This question is neglected here, since the problem is to choose between say 10 and 20 stages, not between 10 and 11.

Moreover, the corresponding gains are related by

$$g^N = G. \tag{14}$$

Hence (11) can be rewritten as

$$\bar{\omega} = m\sqrt{2\Delta}\, N^{-1/2} G^{-1/N}. \tag{15}$$

To find out if an optimum N exists, it is permissible to take N as a continuous variable in the above and equate the derivative to zero. This leads to

$$m\sqrt{2\Delta}\,[-(1/2)N^{-3/2} + (N^{-1/2}/N^2)\ln G]G^{-1/N} = 0$$

or

$$N = 2\ln G$$

or better

$$g = \epsilon^{1/2}. \tag{16}$$

This can be substituted in (15) to give the ultimate bandwidth.

From these considerations, one can state the theorem: *In an identical stage precision resistance-capacitance amplifier with given over-all gain, the ultimate bandwidth is attained when the number of stages is such that the gain per stage is one-half Neper* (=4.33 *decibels or a voltage ratio of* 1.65). *This bandwidth is then given by*

$$\boxed{\bar{\omega} = m(\Delta/\epsilon A)^{1/2}} \tag{17}$$

where A is the over-all amplification in Nepers, i.e.,

$$A = \ln G. \tag{18}$$

Equation (17) no longer exhibits the property of exchangeability of gain and bandwidth. The latter is merely inversely proportional to the square root of the logarithm of the gain. Hence in the ultimate, the bandwidth is a slowly varying function of the gain.

One Shunt-Compensated Stage

In the compensation scheme known as shunt peaking, a small coil L is added in series with r, forming the interstage circuit shown in Fig. 1. Equation (2) is still correct for the relative gain, but z must now be the impedance of the circuit of Fig. 1. ρ can be readily expressed in terms of the circuit parameters, but it takes a more

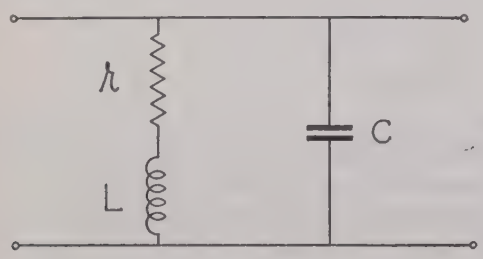


Fig. 1—Interstage circuit of shunt-peaked amplifier.

useful form in terms of the frequency parameter ϕ, defined as before, and the new inductance parameter k defined as

$$k = \omega^0 L/r \tag{19}$$

i.e., as the ratio of coil reactance at the guide frequency to the load resistance. In this language

$$\rho = (1 + jk\phi)/((1 - k\phi^2) + j\phi). \tag{20}$$

Since we are discussing merely the amplitude response, it is sufficient to consider just

$$|\rho|^2 = (1 + k^2\phi^2)/(1 + (1 - 2k)\phi^2 + k^2\phi^4). \tag{21}$$

In the plain resistance-capacitance case, $\phi = 1$ was the half-power point. In (21) if one chooses $k = 1/2$, it is readily seen that the gain is unity at $\phi = 1$. That is a seemingly desirable result. Unfortunately, there arises a slight peak of some 3 per cent in amplitude at about $\phi = 0.69$. If 3 per cent is not an objectionable tolerance, one might say that a bandwidth of $\bar{\omega} = \omega^0$ has been obtained. *But*, if many such stages are used, the 3 per cent grows.

For a reasonably high gain and small tolerance, one can see by using (17) in practical examples that the ultimate bandwidth is determined by the shape of the response curve for $\phi \ll 1$. Hence, for the best ultimate bandwidth, the response should be as flat as possible for *small* values of ϕ. An obvious process for accomplishing this is to equate as far as possible the derivatives of $|\rho|$ with respect to ϕ to zero at $\phi = 0$. This leads directly to the so-called series method for determining parameters, i.e., to the recipe: Equate the top coefficients of the fractional rational function (21) to the corresponding bottom ones, starting with the lowest powers and proceeding upward. That this can be proved readily will be indicated.

If an asterisk designates complex conjugate, one can see from circuit theory that

$$Z(-\omega) = Z^*(\omega)$$

whence,

$$|\rho(-\omega)|^2 = |\rho(\omega)|^2.$$

Hence any $|\rho|^2$ function is *even* in ω, and therefore a function of ϕ^2 only. Since also $|\rho(0)|^2 = 1$ any such $|\rho|^2$ function takes the form of a fractional rational function in ϕ^2 with both constant terms equal to unity. In this form it can be expanded in a convergent power series around the origin whose coefficients are proportional to the derivatives there. By a coefficient comparison, one can show that equating the first n top to the bottom coefficients results in the same number of zero coefficients in the series.

Applying this method to the shunt-peaked function (21), one must satisfy the equation

$$k^2 + 2k - 1 = 0 \tag{22}$$

whence,

$$k = \sqrt{2} - 1 = 0.414 = k_0. \tag{23}$$

A discussion[3] has been going on for some time between the proponents of $k = 1/2$ and $k = 0.414$. Clearly $k = k_0$ gives the flattest curve near the origin, although $k = 1/2$ gives a wider bandwidth for one stage with $\delta < 3$ per cent. Since we are now looking toward best *ultimate* bandwidth, the value k_0 is indicated.

With this choice of k, we seek an explicit expression for the bandwidth $\bar{\omega}$ as a function of the tolerance. In terms of $\bar{\phi}$ corresponding to $\bar{\omega}$, the equation

$$(1 - \delta)^2 = (1 + k_0^2\bar{\phi}^2)/(1 + k_0^2\bar{\phi}^2 + k_0^2\bar{\phi}^4) \tag{24}$$

must hold. The tolerance relationship (10) is involved in this way, since $|\rho|^2$ is a monotone decreasing function for this choice[5] of k.

As before, we make the approximation $\delta \ll 1$, whence

[5] This can be shown by locating the roots of the derivative of (21).

$$\delta = (k_0^2\bar{\phi}^4/2)(1/(1 + k_0^2\bar{\phi}^2 + k_0^2\bar{\phi}^4)).$$

If δ is small enough (so that $\phi < 1$), it is a fair approximation to drop the last factor above, whence

$$g\bar{\omega} = m(2\delta/k_0^2)^{1/4}. \tag{25}$$

This approximation for bandwidth becomes better as δ becomes smaller.

Shunt-Peaked Ultimate

It is interesting to try a similar ultimate computation in this case. Using the same method as in the resistance-capacitance case, one has first

$$\bar{\omega} = m(2\Delta/k_0^2)^{1/4}N^{-1/4}G^{-1/N}. \tag{26}$$

A maximum is found at

$$N = 4A \tag{27}$$

where A is again given by (18). When substituted above, this gives an expression for the ultimate bandwidth, and the theorem: *In an identical stage, shunt-peaked, precision amplifier* ($k = \sqrt{2} - 1$) *with given over-all gain, the ultimate bandwidth is obtained when the number of stages is such that the gain per stage is one-quarter Neper. This bandwidth is then given by*

$$\boxed{\bar{\omega} = m(\Delta/2A\epsilon k_0^2)^{1/4}} \tag{28}$$

The above is clearly larger than the uncompensated ultimate (17), for the quantity in parenthesis is less than unity but greater than before.

Conclusions

The maximum found in (28) is fortunately very broad, since for any appreciable amount of gain, the ultimate design requires an inordinately large number of stages, and is hence not practicable. Its chief role is to determine immediately if given requirements can be met at all. For instance, with ordinary tubes, $g_m = 2500$ micromhos, $c = 25$ micromicrofarads, and with a gain of *5 Nepers* ($= 43.5$ decibels) and a tolerance of 4 per cent, the ultimate bandwidth without compensation is about 860 kilocycles. If this amount of amplification were needed from the camera tube to transmitter, then one can see that modern high-resolution television would be impossible with these tubes without compensation. With compensation under the same conditions, the ultimate bandwidth becomes about 4.8 megacycles.

When a practical problem is given, it is often convenient to compute first the ultimate, and if this is more than enough, then to solve (15) or (26) for the required number of stages.

Similar computations can be carried out for other forms of compensation, for instance for so-called series peaking, but the results in this case are of doubtful value, since the response curve is not monotone. For curves with fluctuations, a different approach is needed.

Equivalent T and Pi Sections for the Quarter-Wavelength Line*

C. G. BRENNECKE†, ASSOCIATE, I.R.E.

Summary—**An equivalent T section is derived for the high-frequency quarter-wavelength line, and the action of the line is shown to follow the behavior of its equivalent network. An equivalent pi section for the line is similarly derived and discussed.**

THE useful properties of the quarter-wavelength electrical transmission line at high frequencies are well known. These properties are customarily deduced by applying the special conditions of the case to the general transmission-line equations. It is the purpose of this discussion to point out that the quarter-wavelength line may be represented by a lumped reactive network of simple structure and interesting behavior, which behavior may then also be expected of the line.

Constants of the Equivalent T

An equivalent T section of the form shown in Fig. 1 may be constructed for any smooth transmission line, if the values of the branch impedances are chosen according to the equations

$$Z_1/2 = Z_0 \tanh \theta/2 \tag{1}$$

$$Z_2 = Z_0/\sinh \theta \tag{2}$$

where $Z_0 = \sqrt{z/y}$ and $\theta = \sqrt{zy}\,l$

Z_0 being the characteristic impedance, θ the "electrical length" of the line in hyperbolic radians, z the series impedance per unit length, l the length of the line, and y the transverse admittance per unit length.

At high frequencies, the series resistance and the transverse conductance of the line become negligible compared to the series reactance and transverse susceptance respectively, and we may write

$$Z_0 = \sqrt{L/C} \tag{3}$$

L being the inductance per unit length and C the capacitance between wires per unit length.

The hyperbolic argument θ may be written in complex form as

$$\theta = \alpha l + j\beta l. \tag{4}$$

The real part αl represents attenuation and may thus

* Decimal classification: R116. Original manuscript received by the Institute; March 18, 1943, revised manuscript received, April 30, 1943.

† Associate Professor of Electrical Engineering, Lehigh University, Bethlehem, Pennsylvania.

be neglected at high frequencies. The quadrature term βl represents phase rotation, which must be $\pi/2$ radians for a quarter-wavelength line. Thus, for this line, substituting the special conditions in (1) and (2),

$$Z_1/2 = \sqrt{L/C}\tanh j\pi/4 = j\sqrt{L/C}\tan \pi/4 = j\sqrt{L/C} \quad (5)$$

$$Z_2 = \sqrt{L/C}/\sinh j\pi/2 = -j\sqrt{L/C}/\sin \pi/2 = -j\sqrt{L/C}. \quad (6)$$

The equivalent T section for the quarter-wavelength

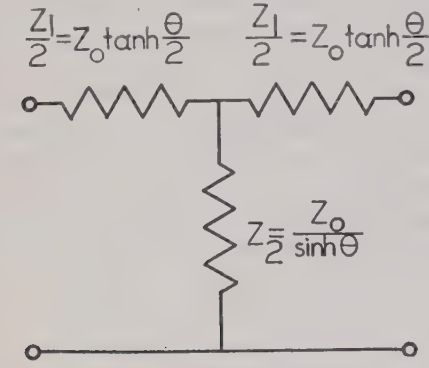


Fig. 1—Equivalent T section.

line then takes the form shown in Fig. 2, with inductances for arms and a capacitance for staff, each having the same value of reactance in ohms at the frequency of operation. This special form of symmetrical T section

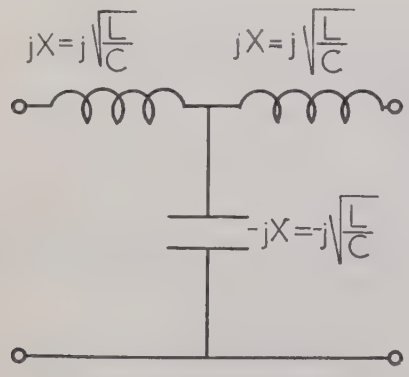


Fig. 2—Inverting T section.

may be called, for want of a better name, the inverting T section, by virtue of a property which may easily be demonstrated.

Properties of the Inverting T Section

Fig. 3 shows a T network of the same type as that of Fig. 2, loaded at its output terminals with an inductive impedance $Z_{34}=R_L+jX_L$. If we call Z_{12} the input impedance of the combination, then

$$\begin{aligned} Z_{12} &= jX+(-jX[R_L+j(X+X_L)]/R_L+jX_L) \\ &= jX+([-jXR_L - X^2+XX_L][R_L-jX_L]/R_L^2+X_L^2) \\ &= (jXR_L^2+jXX_L^2+R_LX^2+R_LXX_L-jXR_L^2 \\ &\quad -XX_LR_L-jX^2X_L-jXX_L^2)/(R_L^2+X_L^2) \\ &= X^2[(R_L/R_L^2+X_L^2)-j(X_L/R_L^2+X_L^2)] \\ &= X^2[G_L-jB_L]=X^2Y_L \end{aligned} \quad (7)$$

where $Y_L=G_L-jB_L$ is the admittance of the load Z_{34}. Thus the insertion of the inverting T section ahead of a load impedance transforms that impedance into a constant times its own admittance. Stated in another way, the impedance R_L+jX_L is transformed into another impedance having an equal phase angle in the opposite sense, and a magnitude determined by the value of the unit reactance X of the T section.

Equation (7) may be restated in the form

$$Z_{12}Z_{34} = X^2 \quad (8)$$

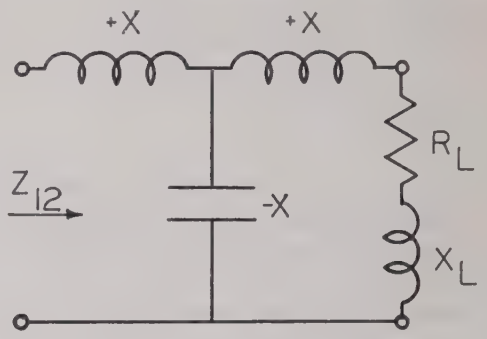


Fig. 3—Loaded inverting T section.

which indicates that the inverting T section has the property of effecting a perfect conjugate match between two impedances, providing only that their phase angles are identical. In the case of two resistances, of course, (8) becomes

$$R_{12}R_{34} = X^2. \quad (9)$$

Equations (8) and (9) are consistent with the conventional theory[1] of resistance matching by means of a T network of pure reactances. The standard equations for such matching are

$$R_{12}R_{34} = -(X_1X_2 + X_2X_3 + X_3X_1) \quad (10)$$

and

$$R_{12}/R_{34} = (X_1 + X_3)/(X_2 + X_3) \quad (11)$$

in which X_1 and X_2 are the arm reactances and X_3 is the staff reactance of the matching network. In the case of the inverting T section $X_1=X_2=X$ and $X_3=-X$, and therefore (10) becomes identical with (9) and (11) becomes indeterminate. It seems to the author that when resistances are to be matched by means of a reactive T network, and no other constraint exists on the values of the components of the network, the use of the inverting T section permits a rapid and easy calculation for the design of the matching net, by means of (6).

In a recently published text,[2] Ware and Reed use an inverting T section to illustrate a calculation in impedance matching. They cite it, however, only as a special case of the symmetrical T section, and make the statement that a symmetrical reactive T section that is, one whose arms are identical but whose staff may be any value of reactance, can be designed to match any

[1] See, for example, W. L. Everitt, "Communication Engineering," McGraw-Hill Book Company, Inc., New York, N.Y., 1937, p. 244 et seq.

[2] L. A. Ware and H. R. Reed, "Communication Circuits," John Wiley and Sons, Inc., New York, 1942. (Reviewed in this issue of the Proceedings, page 54.)

two resistances. This statement is inconsistent with (11), which shows that in general such a symmetrical T section can only match between equal resistances. Since the staff reactance is equal and opposite to the arm reactance in the inverting T section, this constraint is avoided. Of course, as for all reactive matching networks, the match occurs at only one frequency.

It is interesting to note that Fig. 3 may be interpreted as the equivalent circuit of a tuned transformer, with both primary and secondary in resonance before the addition of the load Z_{34}. Since the mutual impedance of the transformer is then $-X$, equation (9) indicates that the coupling is critical; i.e., $Z_m = \sqrt{R_{12}R_{34}}$.

The Equivalent Pi Section

The quarter-wavelength line may also be represented by an equivalent pi section, the constants of which are easily determined. The network of Fig. 4 duplicates

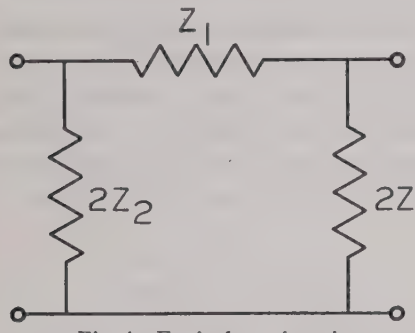


Fig. 4—Equivalent pi section.

the electrical behavior of a transmission line, provided that Z_1 and Z_2 are now impedances specified by the equations

$$Z_1 = Z_0 \sinh \theta \tag{12}$$

$$2Z_2 = Z_0 \coth \theta/2. \tag{13}$$

From (3) and (4) and the special conditions for the quarter-wavelength line at high frequencies,

$$Z_1 = \sqrt{L/C} \sinh j\pi/2 = j\sqrt{L/C} \sin \pi/2 = j\sqrt{L/C} \tag{14}$$

$$2Z_2 = \sqrt{L/C} \coth j\pi/4 = -j\sqrt{L/C} \cot \pi/4 = -j\sqrt{L/C} \tag{15}$$

giving the form shown in Fig. 5 for the equivalent pi section for the quarter-wavelength line, where $X = \sqrt{L/C}$ as before.

The pi section of Fig. (5) may also be derived directly from the T section of Fig. 2 by using the T—pi transformation equations of network theory. Employing the conventional nomenclature for this transformation, if Z_A, Z_B, and Z_C are the elements (taken in clockwise order) of a pi network which is to be electrically equivalent to a T network having arm impedances Z_1 and Z_2 and a staff impedance Z_3, then

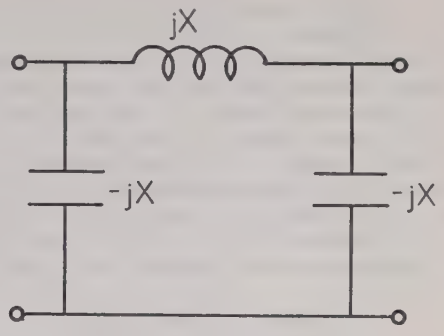


Fig. 5—Inverting pi section.

$$Z_A = (Z_1Z_2 + Z_2Z_3 + Z_3Z_1)/Z_2 \tag{16}$$

$$Z_B = (Z_1Z_2 + Z_2Z_3 + Z_3Z_1)/Z_3 \tag{17}$$

$$Z_C = -(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)/Z_1. \tag{18}$$

Substitution of the values of the components of the T section of Fig. 2 into (16) to (18) will at once demonstrate the equivalence of the pi section of Fig. 5. The latter will thus of course exhibit the same properties of inversion and impedance matching as the original T section. In fact, since individual circuit considerations often require the use of a matching section with capacitive input and output, the inverting pi section may be of more general utility than its T equivalent.

Conclusion

In view of the fact that the quarter-wavelength line may be represented by the T section of Fig. 2 or the pi section of Fig. 5, the properties of these inverting sections may be ascribed to the line. The latter, then, will act as on impedance transformer, giving a perfect match between resistances and a conjugate match between impedances whose phase angles are the same. It will also invert an impedance, making it appear as a constant times its own admittance.

While no previously unknown property of the quarter-wavelength line has been shown here, it may be that a similar representation of other high-frequency lines, stubs, and antenna feeders by equivalent pi or T sections will lead to a better appreciation of their potential applications.

The Receiving Antenna*

RONOLD KING†, SENIOR MEMBER, I.R.E., AND CHARLES W. HARRISON, JR.†, ASSOCIATE, I.R.E.

Summary—After a discussion of the general problem of coupled antennas, the distribution of current in a center-loaded receiving antenna with its axis in the plane of a linearly polarized electric field is derived. Distribution curves are shown for a wide range of lengths, thicknesses, and load impedances. A simple "equivalent" circuit is obtained for the loaded antenna for determining the current in the load. The effective length for a receiving antenna is defined and curves are shown for a wide range of lengths and several thicknesses. An expression for the power transferred to a matched load is derived and curves for computing it are given for antennas over the same range of lengths and thicknesses. Optimun conditions are discussed.

General Introduction

IN A recent paper,[1] the distribution of current along a cylindrical center-driven antenna was analyzed. In the present paper the distribution of current along a loaded receiving antenna which is in the far zone of a transmitter will be considered in detail. This is a relatively simple special case of the general problem of the distribution of current along two coupled antennas which will be formulated first. The formal solution will then be limited by the particular conditions for the receiving antenna, the detailed analysis of the general case being reserved for a later paper.

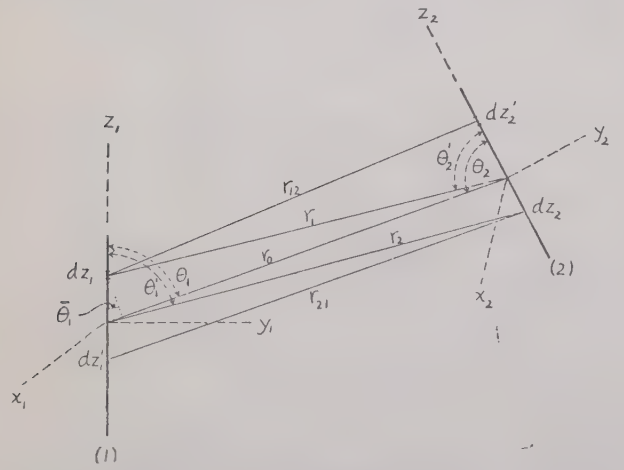


Fig. 1—Orientation of the elements dz' (along the axis) and dz (on the surface) for two antennas.

Consider two straight cylindrical antennas which are oriented in space in an arbitrary way as shown in Fig. 1. Let the origin of a co-ordinate system x_1, y_1, z_1 or R_1, Θ_1, Φ_1 be located at the center of antenna 1; let a second system of co-ordinates x_2, y_2, z_2 or R_2, Θ_2, Φ_2 be located at the center of antenna 2. Whenever convenient the subscript 1 on the co-ordinate system of antenna 1 will be omitted. In each case the z axis coincides with the axes of the antenna. As previously explained[1] the continuity of the tangential component of the electric field along the cylindrical envelope of each antenna leads in this case to

$$E_{1z}{}^{i} = E_{1z}{}^{0} \tag{1a}$$
$$E_{2z}{}^{i} = E_{2z}{}^{0}. \tag{1b}$$

Upon introducing the internal impedance z^i and the total current I for each conductor and making use of the definition of the vector potential one has, just as before,

$$(\partial/\partial z_1)(\text{div } A_1{}^{0}) + \beta^2 A_{z_1}{}^{0} = j(\beta^2/\omega) z_1{}^{i} I_1 \tag{2a}$$
$$(\partial/\partial z_2)(\text{div } A_2{}^{0}) + \beta^2 A_{z_2}{}^{0} = j(\beta^2/\omega) z_2{}^{i} I_2. \tag{2b}$$

But in the present case $A_1{}^0$ is the vector potential on the surface of antenna 1 due to currents not only in antenna 1 but also in antenna 2. Similarly $A_2{}^0$ is the vector potential on the surface of antenna 2 due to I_2 and I_1. Thus,

$$A_1{}^{0} = A_{11}{}^{0} + A_{12}{}^{0} \tag{3a}$$
$$A_2{}^{0} = A_{22}{}^{0} + A_{21}{}^{0}. \tag{3b}$$

Here $A_{11}{}^0$ is the vector potential on the surface of antenna 1 due to I_1; $A_{12}{}^0$ is the vector potential on the surface of antenna 1 due to I_2; $A_{22}{}^0$ is the vector potential on the surface of antenna 2 due to I_2; $A_{21}{}^0$ is the vector potential on the surface of antenna 2 due to I_1. They are given by

$$A_{11}{}^{0} = \frac{\Pi}{4\pi}\int_{-h_1}^{h_1} I_1' \frac{e^{-j\beta r_{11}}}{r_{11}}\, dz_1' \tag{4a}$$
$$A_{12}{}^{0} = \frac{\Pi}{4\pi}\int_{-h_2}^{h_2} I_2' \frac{e^{-j\beta r_{12}}}{r_{12}}\, dz_2' \tag{4b}$$
$$A_{22}{}^{0} = \frac{\Pi}{4\pi}\int_{-h_2}^{h_2} I_2' \frac{e^{-j\beta r_{22}}}{r_{22}}\, dz_1' \tag{4c}$$
$$A_{21}{}^{0} = \frac{\Pi}{4\pi}\int_{-h_1}^{h_1} I_1' \frac{e^{-j\beta r_{21}}}{r_{21}}\, dz_1'. \tag{4d}$$

The following notation has been used:

$\beta = 2\pi/\lambda$; $\Pi = 4 \times 10^{-7}$ henry per meter

$$r_{11} = \sqrt{(z_1 - z_1')^2 + a_1^2} \tag{5a}$$
$$r_{22} = \sqrt{(z_2 - z_2')^2 + a_2^2} \tag{5b}$$
r_{12} is the distance from dz_1 to dz_2' (5c)
r_{21} is the distance from dz_1' to dz_2. (5d)
a_1 is the radius of antenna 1
a_2 is the radius of antenna 2.

If the two antennas are in the far zone with respect to each other, and r_0 is the distance between their centers so that one can write

$$\beta r_0 \gg 1; \quad r_0 \gg h_1; \quad r_0 \gg h_2, \tag{6}$$

* Decimal classification: R120. Original manuscript received by the Institute, February 16, 1943; revised manuscript received, June 2, 1943.

† Cruft Laboratory and the Research Laboratory of Physics, Harvard University, Cambridge, Mass.

[1] Ronold King and Charles W. Harrison, Jr., "The distribution of current along a symmetrical center-driven antenna," PROC. I.R.E., vol. 31, pp. 548–567; October, 1943.

(4b) and (4d) reduce to

$$A_{12}{}^0 = \frac{\Pi}{4\pi}\frac{e^{-i\beta r_1}}{r_1}\int_{-h_2}^{h_2} I_2' e^{i\beta z_2' \cos\theta_2'} dz_2' = \frac{\Pi}{4\pi}\frac{e^{-i\beta r_1}}{r_1} F(I_2, \theta_2). \quad (7a)$$

$$A_{21}{}^0 = \frac{\Pi}{4\pi}\frac{e^{-i\beta r_2}}{r_2}\int_{-h_1}^{h_1} I_1' e^{-i\beta z_1' \cos\theta_1'} dz_1' = \frac{\Pi}{4\pi}\frac{e^{-i\beta r_2}}{r_2} F(I_1, \theta_1). \quad (7b)$$

Here r_1 is the distance from the center of antenna 2 to the element dz_1 in antenna 1 and r_2 is the distance from the center of antenna 1 to an element dz_2 of antenna 2 as shown in Fig. 1. θ_1' is the angle between the z_1 axis and r_2, θ_2' is the angle between the z_2 axis and r_1, θ_1 is the angle between the z_1 axis and r_0, and θ_2 is the angle between the z_2 axis and r_0.

Differential Equation for the Unloaded Receiving Antenna

Let antenna 1 be a receiving antenna consisting of a straight cylindrical conductor with no impedance at its center. Let it be in the far zone with respect to a transmitting antenna 2. The equation (2a) applies with $A_{11}{}^0$ given by (4a) and $A_{12}{}^0$ given by (7a).

One can write

$$A_{12}{}^0 = K_{12}e^{-i\beta r_1}. \quad (8a)$$

From (7a) one has

$$K_{12} = (\Pi/4\pi)F(I_2\theta_2)/r_1). \quad (8b)$$

K_{12} is practically constant over the length of antenna 1. It has been shown[1] that the vector potential due to the current in an antenna is directed parallel to that antenna except at points very near the small end faces. Accordingly $A_{11}{}^0$ has only the z_1 component $A_{11z}{}^0$. Using this value, and with (8a) substituted in (2a) according to (3a),

$$\{(\partial^2 A_{11z}{}^0/\partial z_1{}^2) + \beta^2 A_{11z}{}^0\} + \{(\partial/\partial z_1)[\mathrm{div}_1 (K_{12}e^{-i\beta r_1})] + \beta^2 K_{12z}e^{-i\beta r_1}\} = j(\beta^2/\omega)z_1{}^i I_1. \quad (9)$$

The operation div_1 involves only the co-ordinates of antenna 1 with respect to which K_{12} is sensibly constant. Accordingly div K_{12} vanishes and, writing s for r_1,

$$\mathrm{div}_1 [K_{12}e^{-i\beta r_1}] = (K_{12}, \mathrm{grad}_1 e^{-i\beta s}) = (K_{12}, s_1)[-j\beta e^{-i\beta s}]. \quad (10)$$

Since both K_{12} and the unit vector s_1, directed along $s=r_1$, are independent of differentiation with respect to z_1,

$$(\partial/\partial z_1)[\mathrm{div}_1 (K_{12}e^{-i\beta s})] = (K_{12}, s_1)\{-\beta^2 e^{-i\beta s}\}(\partial s/\partial z_1). \quad (11)$$

With (8a), and noting that

$$(\partial s/\partial z_1) = \sin\bar{\theta}_1, \quad (12)$$

where $\bar{\theta}_1$ is the complement of θ_1 as shown in Figs. 1 and 2,

$$(\partial/\partial z_1)[\mathrm{div}_1 (K_{12}e^{-i\beta s})] = -\beta^2 A_{12s}{}^0 \sin\bar{\theta}_1. \quad (13)$$

Here,

$$A_{12s}{}^0 = (K_{12}, s_1)e^{-i\beta s} \quad (14)$$

is the component of $A_{12}{}^0$ parallel to the line $s=r_1$.

The general equation (9) may now be written as follows:

$$(\partial^2 A_{11z}{}^0/\partial z_1{}^2) + \beta^2 A_{11z}{}^0 + \beta^2(A_{12z}{}^0 - A_{12s}{}^0 \sin\bar{\theta}_1) = j(\beta^2/\omega)z_1{}^i I_1. \quad (15)$$

Since the distance $r_1=s$ is measured from the center of the transmitting antenna 2 to any element dz_1 along the axis of antenna 1, it is a function of z_1. Thus, for antenna 1 in the far zone with respect to antenna 2 one has (see Fig. 1)

$$r_1 = s = r_0 - z_1 \sin\bar{\theta}_1. \quad (16)$$

Here $\bar{\theta}_1$ is the angle between the axis of antenna 1 and a perpendicular dropped from dz_1 on to r_0 as shown in Figs. 1 and 2.

Thus $$A_{12}{}^0 = K_{12}e^{-i\beta s} = K_{12}e^{-i\beta r_0}e^{+i\beta z_1 \sin\bar{\theta}_1}. \quad (17)$$

This may be rearranged as follows:

$$A_{12}{}^0 = \tfrac{1}{2}[A_{12}{}^0(z) + A_{12}{}^0(-z)] + \tfrac{1}{2}[A_{1z}{}^0(z) - A_{12}{}^0(-z)] \quad (18)$$

with

$$\tfrac{1}{2}[A_{12}{}^0(z) + A_{12}{}^0(-z)] = K_{12}e^{-i\beta r_0}\cos(\beta z_1 \sin\bar{\theta}_1) \quad (18a)$$

$$\tfrac{1}{2}[A_{12}{}^0(z) - A_{12}{}^0(-z)] = jK_{12}e^{-i\beta r_0}\sin(\beta z_1 \sin\bar{\theta}_1). \quad (18b)$$

The relation (18a) implies the following symmetry relations:

$$A_{12}{}^0(z) = A_{12}{}^0(-z); \qquad I(z) = I(-z). \quad (19a)$$

The relation (18b) satisfies the symmetry conditions:

$$A_{12}{}^0(z) = -A_{12}{}^0(-z); \quad I(z) = -I(-z). \quad (19b)$$

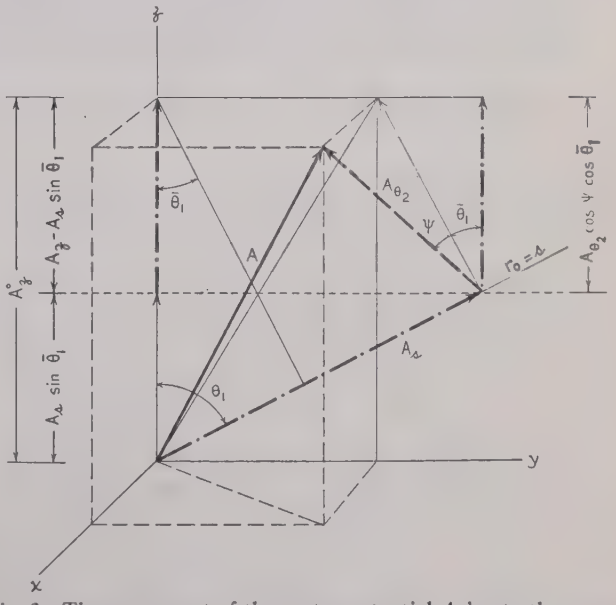


Fig. 2—The component of the vector potential A due to the current in a distant antenna (2) along antenna 1 when this is placed along the z axis shown in the figure.

Each of the two parts in (18), as written, respectively, in (18a) and (18b), leads to an independent solution for the current. The general solution is the superposition of the two. It is to be noted that the distribution of current that is obtained using (18a) and which obeys the symmetry relation for current in (19a) is one in which the current is unidirectional through the center of the antenna. If a load is placed at the center a potential

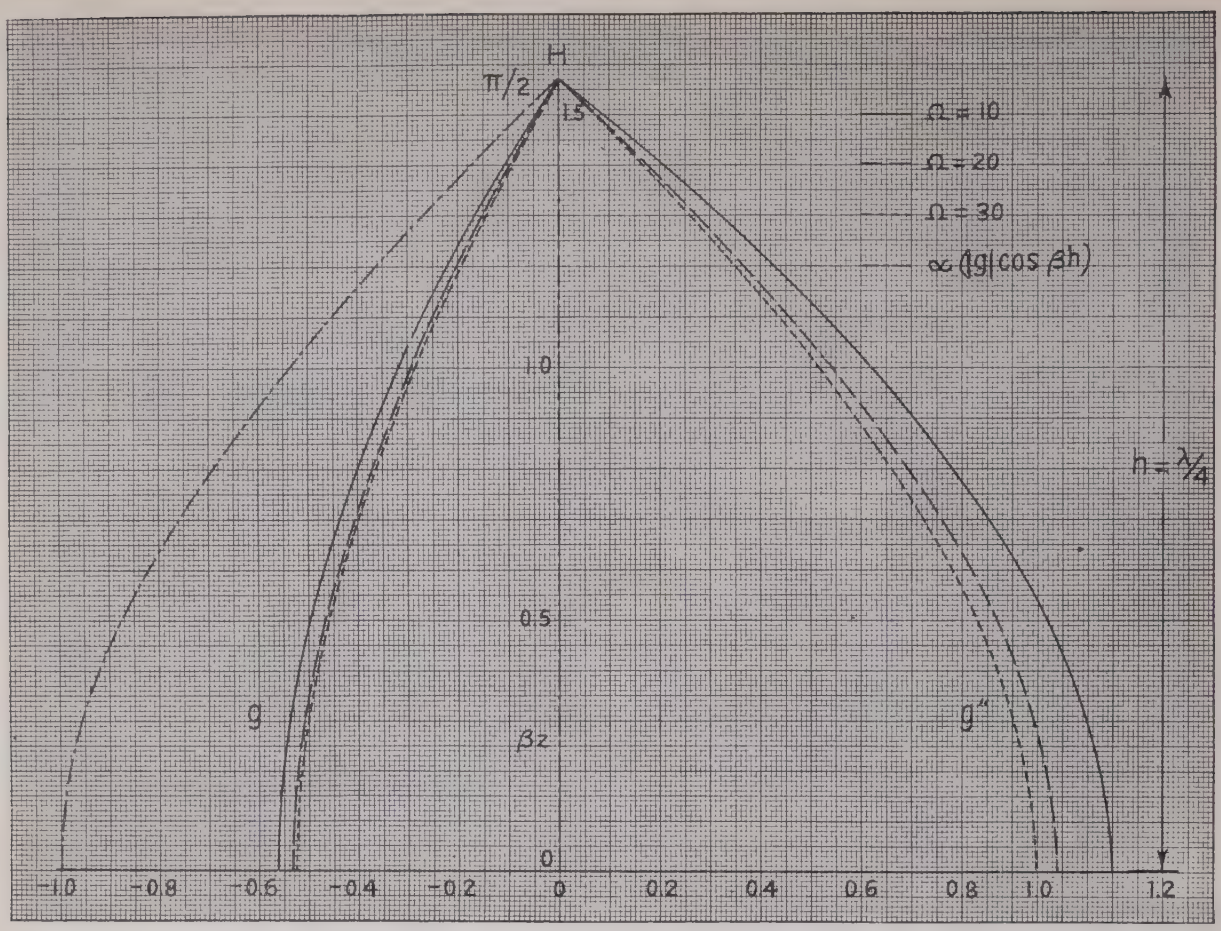


Fig. 3—The functions g' and g'' for an unloaded receiving antenna of half length $h=\lambda/4$. g'' is proportional to the component of current in phase with the electric field, g' is proportional to the component in phase quadrature with the field if this is parallel to the antenna.

difference can be established across it. On the other hand the distribution that is obtained from (18b) and which obeys (19b) is one in which the currents from the two halves flow simultaneously toward or away from the center reducing to zero at the center. Such a current can establish no potential difference across a load which is symmetrically placed at the center. If the load is not placed at the center of a symmetrical antenna the current due to (18b) is also significant in maintaining a potential difference across it. When the load is introduced in the following analysis it will be assumed that it is placed at the center of the antenna so that only (18a) is of importance in determining the current in it and the power transferred to it. Accordingly from the point of view of this load at the center one may write (18a) for $A_{12}{}^0$ in (15). For convenience in writing (18a) will be denoted by $A_{12}{}^0$. Thus for the present analysis let

$$A_{12}{}^0 = \tfrac{1}{2}[A_{12}{}^0(z_1) + A_{12}{}^0(-z_1)] = K_{12}e^{-j\beta r_0}\cos(\beta z_1 \sin\theta_1), \quad (20)$$

with the understanding that (18b) has been omitted. It is important to note, however, that the distribution of current along the antenna obtained using (20) is, in general, not the entire current at other points than the center. Distribution curves computed below give only the components of current due to (18a), but since they are computed only for the special case for which $\sin\bar{\theta}_1=0$, so that (18b) vanishes identically, they actually represent the entire current. Upon substituting (20) in (15) with the notation

$$q = \beta \sin\bar{\theta}_1 \quad (21)$$

one has

$$(\partial^2 A_{11z}/\partial z_1{}^2) + \beta^2 A_{11z} = (j\beta^2/\omega)z_1{}^i I_{1z} - \beta^2 e^{-j\beta r_0}\cos(qz_1)[K_{12z} - K_{12z}\sin\bar{\theta}_1]. \quad (22)$$

Let the following shorthand notation be introduced with (17):

$$U \equiv -(j\omega(A_{12z}{}^0 - A_{12z}{}^0\sin\bar{\theta}_1)/\beta\cos^2\bar{\theta}_1)_{z_1=0} \quad (23a)$$

or

$$U \equiv -j\omega(K_{12z} - K_{12z}\sin\bar{\theta}_1)e^{-j\beta r_0}/\beta\cos^2\bar{\theta}_1. \quad (23b)$$

The components of the vector potential in (23) *are taken at* $z_1=0$, *so that the function* U *is referred to the center of the antenna.* Here U is dimensionally a potential difference measured in volts. With this notation and (21), (22) may be written as follows:

$$(\partial^2 A_{11z}/\partial z_1{}^2) + \beta^2 A_{11z} = j(\beta^2/\omega)[I_{1z}z^i - \beta U(1-(q^2/\beta^2))\cos qz_1]. \quad (24)$$

A formally similar nonhomogeneous wave equation was derived by Hallén[2] following a somewhat different method.

The function U defined by (23) may be expressed in a

[2] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nova Acta*, Royal Soc. Sciences, (Uppsala) vol. 11, pp. 1–44; November, 1938.

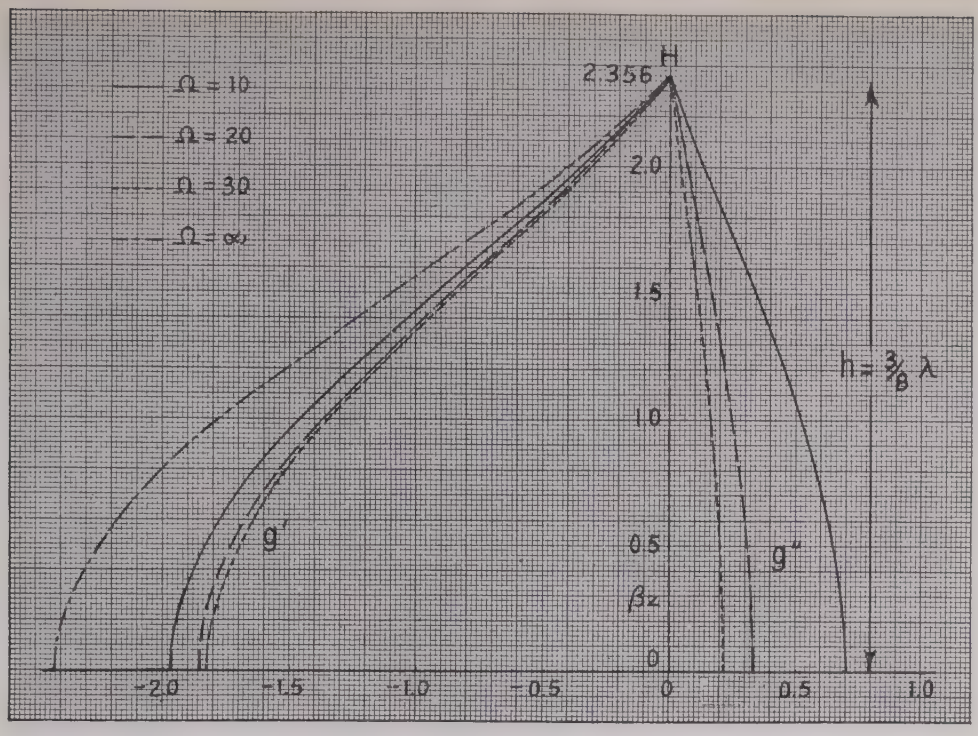


Fig. 4—The functions g' and g'' for an unloaded receiving antenna of half-length $h=3\lambda\backslash 8$.

different way. Since the receiving antenna 1 is in the far zone of the transmitting antenna 2 the entire far-zone electric field may be derived from (7a) according to

$$E_{12\theta_2} = -j\omega A_{12\theta_2} = \frac{j\omega\Pi}{4\pi}\frac{e^{-i\beta r_1}}{r_1}\int_{-h^2}^{h_2} I_2' {}^{i\beta z_2' \cos\theta_2'} \sin\theta_2' dz_2'. \quad (25)$$

Here the components $E_{12\theta_2}$ and $A_{12\theta_2}$ are directed tangent to a great sphere about the transmitting antenna 2 along the spherical co-ordinate θ_2 with origin at the center of antenna 2. They are necessarily perpendicular to a radius vector drawn from this origin in any direction, in particular, perpendicular to a line $r_1=s$ drawn to any point on antenna 1. Because s is large all such lines are practically parallel to each other and to r_0. The component of A_{12} perpendicular to s is shown in Fig. 2. (The subscripts 12 on $A_{12\theta_2}$ have been omitted in Fig. 2. They will be omitted below.) It is clear from the figure that

$$(A_{12z}{}^0 - A_{12s}{}^0 \sin\bar{\theta}_1)_{z_1=0} = (A_{\theta_2}{}^0 \cos\psi \cos\bar{\theta}_1)_{z_1=0}. \quad (26)$$

Accordingly, with (25) and (23a),

$$U = -j\omega(A_{\theta_2}{}^0 \cos\psi/\beta \cos\bar{\theta}_1)_{z_1=0} = (E_{\theta_2}\cos\psi/\beta\cos\bar{\theta}_1)_{z_1=0}. \quad (27)$$

(The subscripts 12 have been omitted from $E_{12\theta_2}$.) This is the form introduced by Hallén. It is often more convenient than (23) because (25) is usually more readily evaluated, and also because E_{θ_2} may be measured experimentally.

It is important to note that when the receiving antenna 1 lies in a plane perpendicular to the line r_0 joining its center to the distant transmitter that it is also perpendicular to $r_1=s$, so that, from Fig. 2,

$$\bar{\theta}_1 = 0; \qquad \cos\bar{\theta}_1 = 1 \quad (28)$$

and from (21) and (27),

$$q = 0; \qquad U = (E_{\theta_2}\cos\psi/\beta)_{z_1=0}. \quad (29)$$

Formal Solution of the Differential Equation

The nonhomogeneous wave equation (24) may be solved for $A_{11z}{}^0$ in the same way as has been shown.[1] Omitting the subscript 1 on z_1 throughout, the complementary function is exactly as before.

$$A_c{}^0 = -j/c[C_1\cos\beta z + C_2\sin\beta|z|] \quad (30)$$

after taking into account the symmetry condition (18). The particular integral includes a term in U in addition to that previously given with z^i as a factor. It is

$$A_p{}^0 = \frac{j}{c}\left[z^i\int_0^z I(s)\sin\beta(z-s)ds - U\cos qz\right]. \quad (31)$$

It is readily verified by direct differentiation that (31) satisfies (24). Accordingly the general solution of (24) is

$$A_{11z}{}^0 = \frac{-j}{c}\left\{C_1\cos\beta z + C_2\sin\beta|z| - z^i\int_0^z I(s)\sin\beta(z-s)ds + U\cos qz\right\}. \quad (32)$$

If the antenna were driven at the center by a driving potential $V_0{}^e$, the arbitrary constant C_2 would have the value $\frac{1}{2}V_0{}^e$. Since $V_0{}^e$ is zero for the unloaded receiving antenna, C_2 vanishes, and the solution simplifies to

$$A_{11z}{}^0 = \frac{-j}{c}\left\{C_1\cos\beta z - z^i\int_0^z I(s)\sin\beta(z-s)ds + U\cos qz\right\}. \quad (33)$$

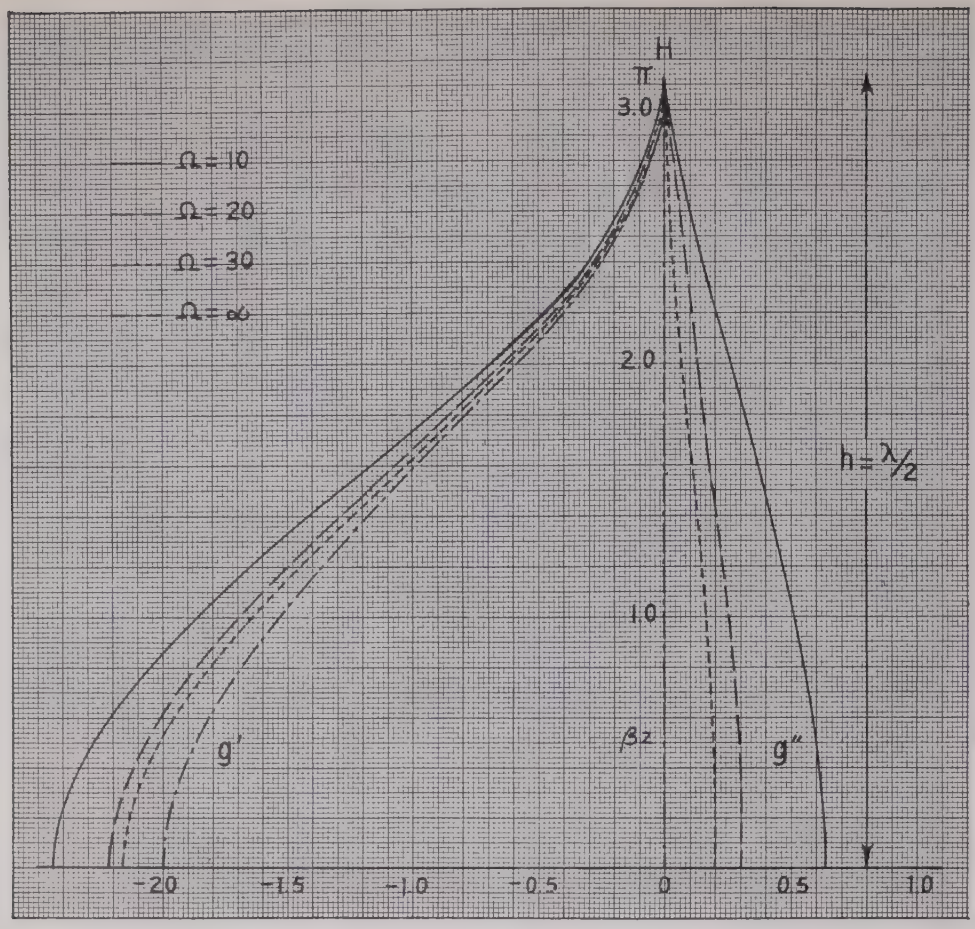


Fig. 5—The functions g' and g'' for an unloaded receiving antenna of half-length $h=\lambda/2$.

Upon introducing the integral (4a) for $A_{11z}{}^0$ in (33) and expanding this exactly as before,[1] one obtains an expression for I_{1z} like (35) of reference 1 but with $U\cos qz$ appearing in place of $\frac{1}{2}V_0{}^e\sin\beta|z|$. The value of I_{1z} vanishes at the ends where $z=h$. By setting $z=h$ one obtains an expression like (36) in reference 1 but with $U\cos qh$ appearing instead of $\frac{1}{2}V_0{}^e\sin\beta h$. The final solution for I_z is exactly like (38), reference 1, with $U(\cos qh\cos\beta z-\cos qz\cos\beta h)$ written instead of $\frac{1}{2}V_0{}^e(\sin\beta|z|-\sin\beta h)$. The same result was obtained by Hallén.

By applying exactly the same method of successive approximations in powers of $1/\Omega$ where, as before,

$$\Omega = 2\ln(2h/a) \tag{34}$$

with a the radius and $2h$ the length of the antenna, one obtains the following formula for I_z with all terms involving higher powers of $1/\Omega$ assumed negligible:

$$I_z = \frac{j4\pi U}{\Omega R_c}\left\{\frac{[\cos qh\cos\beta z-\cos qz\cos\beta h]+(1/\Omega)[m_1{}^{\mathrm{I}}+jm_1{}^{\mathrm{II}}]}{\cos\beta h+(1/\Omega)[A_1{}^{\mathrm{I}}+jA_1{}^{\mathrm{II}}]}\right\}. \tag{35}$$

Here $R_c\equiv\sqrt{\Pi/\Delta}=376.7$ ohms. Just as before,

$$A_1{}^{\mathrm{I}}+jA_1{}^{\mathrm{II}}=F_1(h) \tag{36}$$

while,

$$m_1{}^{\mathrm{I}}+jm_1{}^{\mathrm{II}}=F_1(z)\cos qh-F_1(h)\cos qz+S_1(h)\cos\beta z-S_1(z)\cos\beta h. \tag{37}$$

The functions $S_1(z)$ and $S_1(h)$ are given in the Appendix. The functions $F_1(z)$ and $F_1(h)$ may be found in Appendix II of reference 1. For convenience let the numerator in (35) be denoted by

$$n^{\mathrm{I}}+jn^{\mathrm{II}}=ne^{j\psi_n} \tag{38}$$

with $n^{\mathrm{I}}=\cos\beta z-\cos\beta h+m_1{}^{\mathrm{I}}/\Omega;\ n^{\mathrm{II}}=m_1{}^{\mathrm{II}}/\Omega;$ (39)

and $n=\sqrt{(n^{\mathrm{I}})^2+(n^{\mathrm{II}})^2};\ \psi_n=\tan^{-1}(n^{\mathrm{II}}/n^{\mathrm{I}}).$ (40)

Similarly let the denominator in (35) be denoted

$$D^{\mathrm{I}}+jD^{\mathrm{II}}=De^{j\psi_D} \tag{41}$$

with $D^{\mathrm{I}}=\cos\beta h+A_1{}^{\mathrm{I}}/\Omega;\quad D^{\mathrm{II}}=A_1{}^{\mathrm{II}}/\Omega$ (42)

and $D=\sqrt{(D^{\mathrm{I}})^2+(D^{\mathrm{II}})^2};\ \psi_D=\tan^{-1}(D^{\mathrm{II}}/D^{\mathrm{I}}).$ (43)

Also let $g'=n\cos(\psi_D-\psi_n)$ (44)

$$g''=n\sin(\psi_D-\psi_n). \tag{45}$$

Here g' and g'' are functions of z, whereas D is not. With this notation (35) becomes

$$I_z=(4\pi U/\Omega R_cD)(g''+jg')=(U/30\Omega D)(g''+jg'). \tag{46}$$

Thus g'' is proportional to the component of current in phase with U and hence with the electric field at the center of the antenna; g' is proportional to the component of current in phase quadrature with U and

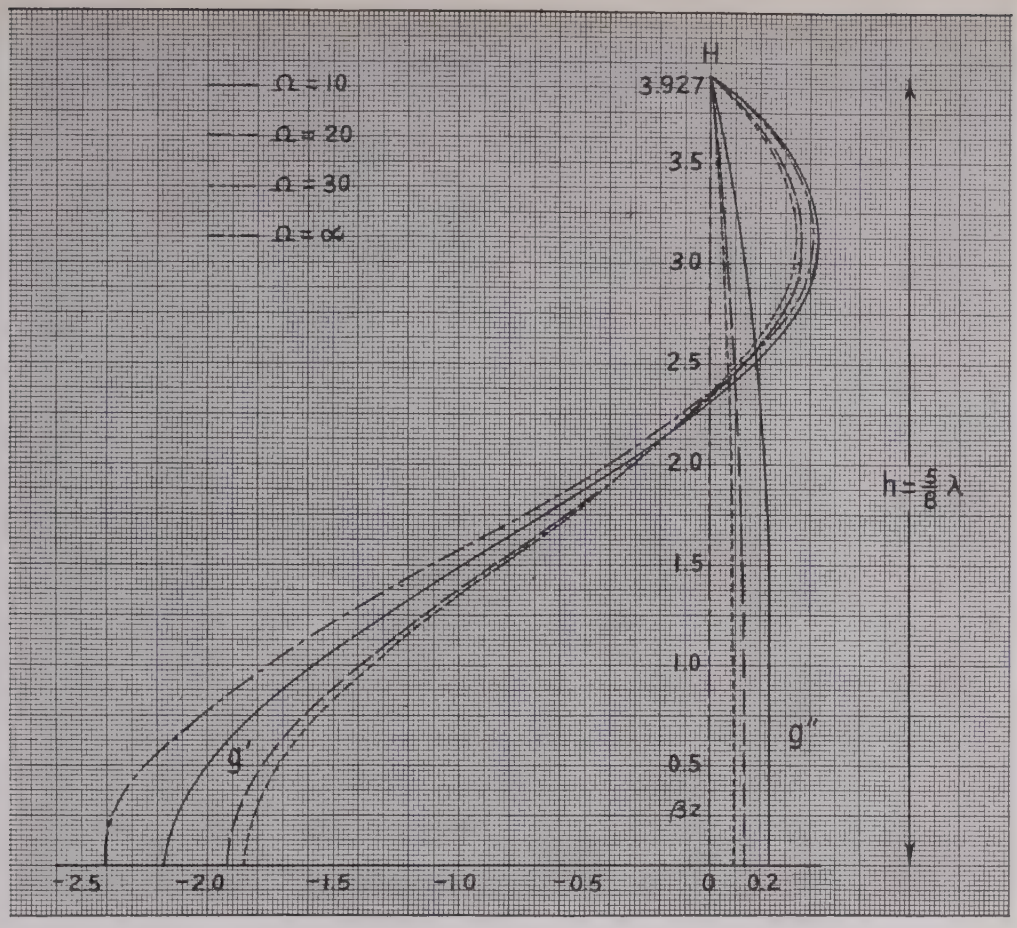


Fig. 6—The functions g' and g'' for an unloaded receiving antenna of half-length $h=5\lambda/8$.

the field. In amplitude-phase-angle form one has

$$I_z = (U/30\Omega D)\,|\,g\,|\,e^{j\theta} \tag{47}$$

$$|\,g\,| = \sqrt{(g'')^2 + (g')^2};\quad \theta = \tan^{-1}(g'/g''). \tag{48}$$

Numerical Solution for a Receiving Antenna in a Plane of Constant Phase of a Linearly Polarized Field

In carrying out numerical computations using (46) and (47) only the important special case of a receiving antenna in the plane of constant phase of a linearly polarized electric field will be considered.[3] This case is defined by (28) and the simplifying consequences are contained in (29). Subject to (28) the complicated function given in (37) reduces to the relatively very simple form,

$$m_1^{\mathrm{I}} + jm_2^{\mathrm{II}} = F_1(z) - F_1(h) \tag{49}$$

Curves showing the functions g'' and g' for use with formula (46) have been computed subject to (28). They are shown in Figs. 3 to 6 for the same lengths and thicknesses previously calculated for the driven antenna. (Reference 1, Figs. 7 to 10.) Curves giving $\sqrt{(g'')^2+(g')^2}$ and $\theta=\tan^{-1}(g'/g'')$ also subject to (28) are given in Figs. 7 to 10. Values of the factor $1/30\Omega D$ for the several cases are shown in Table I. Figs. 3 to 10 with Table I completely characterize the distribution of current along a highly conducting, straight, cylindrical antenna of radius a and half-length h placed in the radiation zone of a transmitting antenna and oriented so that it lies in a plane perpendicular to the line joining it to the transmitter; i.e., the antenna must lie in a plane of constant phase of a linearly polarized electric field. A comparison of these figures with the corresponding ones for a driven antenna as given[1] reveals that *the distributions of current in the two cases are completely different.* If account is taken of the factors $1/30\Omega D$ or $1/60\Omega D$ the amplitude of the current for a constant applied voltage in the driven case and for a given electric field in the receiving case is seen to increase as the ratio

TABLE I

$\frac{h^*}{\lambda}$	$H=\beta h$ (rad)	$\frac{1}{30\Omega D}$		$\frac{1}{60*\Omega D}$	
		Ω=10	Ω=30	Ω=10	Ω=30
0.25	1.571	23.64×10^{-3}	23.62×10^{-3}	7.54×10^{-3}	7.53×10^{-3}
0.375	2.356	4.64	1.57	1.48	0.50
0.50	3.141	3.64	1.18	1.16	0.376
0.625	3.927	5.61	1.66	1.79	0.529

* h is the half-length of a symmetrical center-loaded antenna; a is its radius; $\Omega\equiv2\ \ln\ (2h/a)$.

[3] Important aspects of the more general case are described in a companion to this paper entitled, "The receiving antenna in plane-polarized field of arbitrary orientation," Proc. I.R.E., this issue, pp. 35–50.

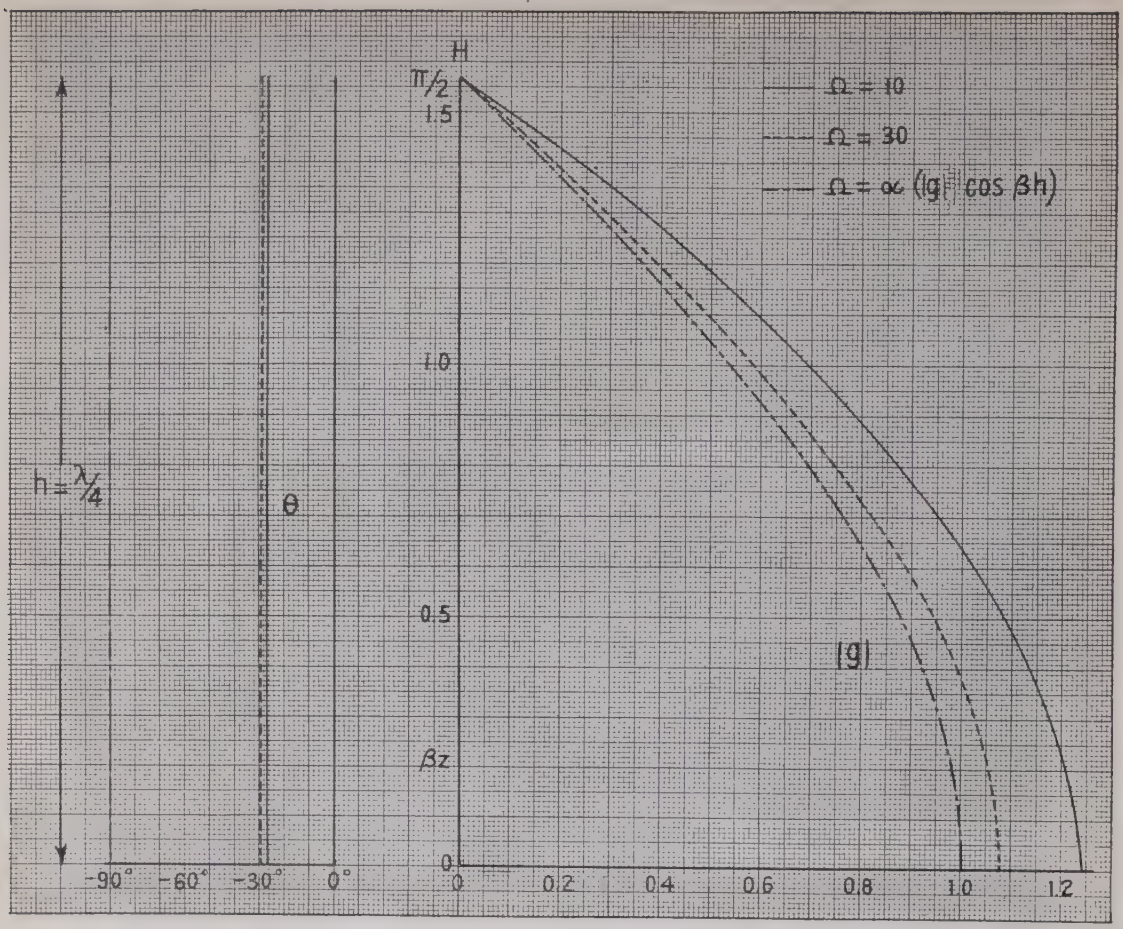


Fig. 7—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $=\lambda/4$; $|g|$ is proportional to the magnitude of the current; θ is its angle referred to the electric field.

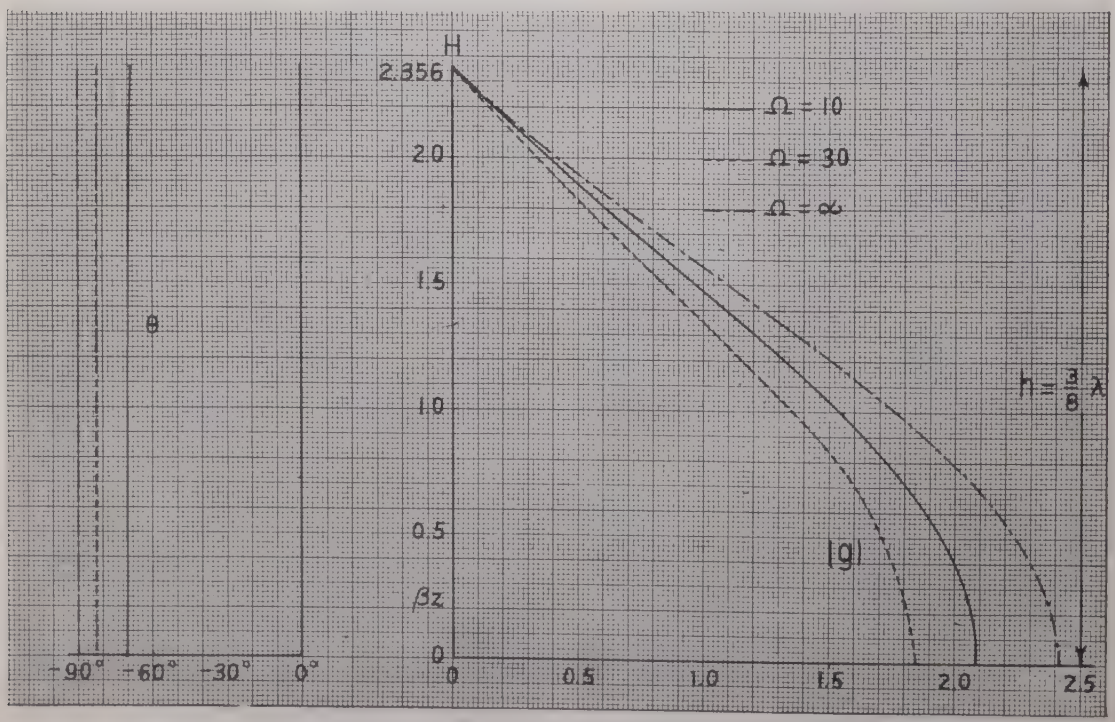


Fig. 8—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=3\lambda/8$.

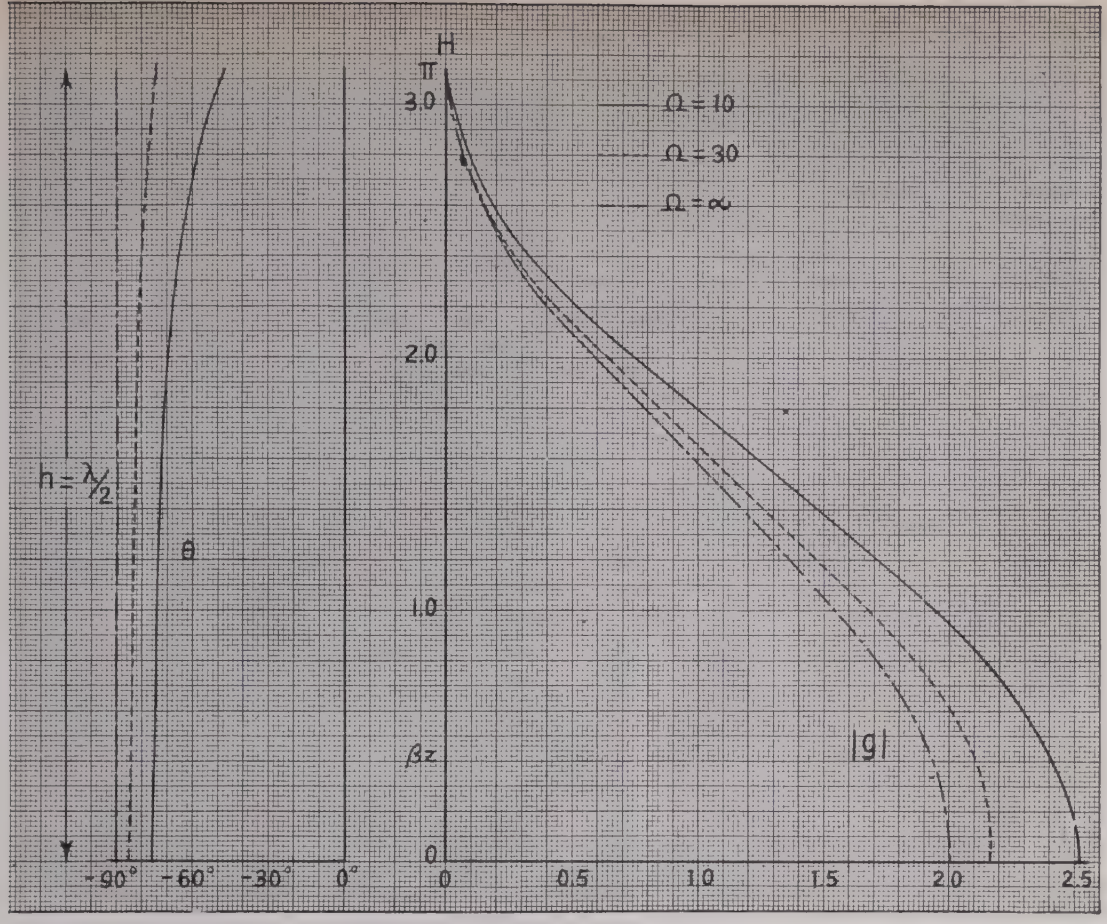


Fig. 9—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=\lambda/2$.

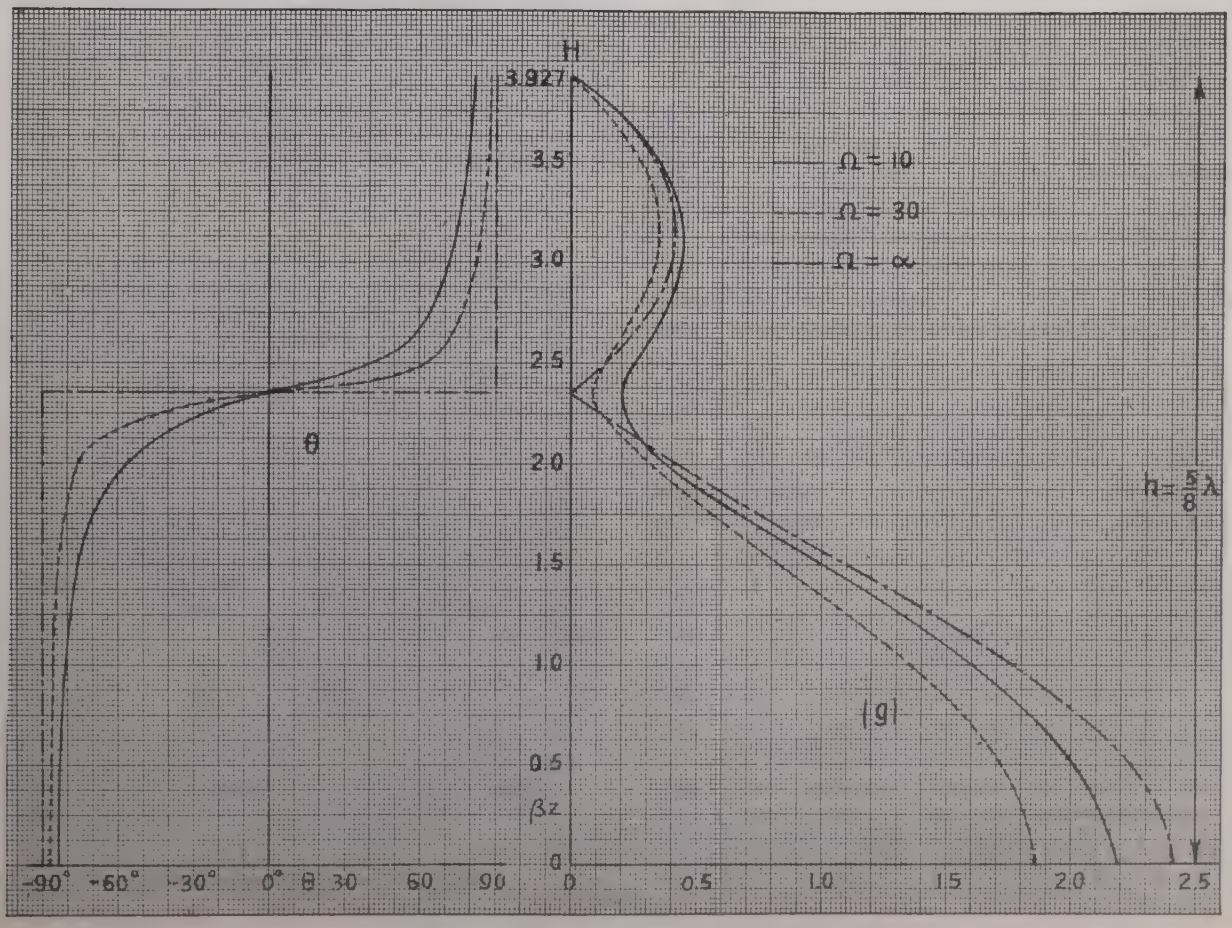


Fig. 10—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=5\lambda/8$.

of h/a in $\Omega = 2\ln(2h/a)$ is made smaller by increasing the radius a. This may be expressed in terms of a decrease in impedance between two sufficiently closely spaced terminals in the driven antenna. In the receiving antenna an impedance in the conventional sense cannot be defined because the externally applied driving forces are distributed along the entire length of the antenna and not concentrated between two terminals which are

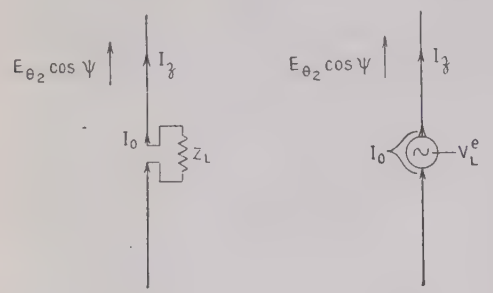


Fig. 11—"Equivalent" circuit for a loaded receiving antenna.

well within the near zone with respect to each other. It will be shown below that the receiving antenna can be replaced by an "equivalent" circuit from the point of view of the current at its center in which a fictitious lumped electromotive force is defined. It is important to note that the current in the receiving antenna is directly proportional to the factor

$$U \text{ (volts per wave number)} = (E_{\theta_2}\cos\psi)/\beta = (\lambda E_{\theta_2}\cos\psi)/2\pi. \quad (50)$$

Here $E_{\theta_2}\cos\psi$ is the component of the electric field due to the distant transmitting antenna (number 2) along the axis of the receiving antenna provided this lies in a plane of constant phase of the linearly polarized electric field. The factor U is measured in volts per wave number, where the wave number is the number of wavelengths per meter. The more general case which does not satisfy condition (28) must be treated separately. This has been carried out completely by Hallén[2] only for the limiting case of an infinitely thin antenna with matched load. It is considered in detail in a companion paper.[3]

The Distribution of Current for an Extremely Thin Receiving Antenna

The approximate distribution of current along an extremely thin receiving antenna is obtained from (35) by neglecting the terms in $1/\Omega$ in both numerator and denominator. With (29) it is

$$I_z = (j4\pi U/\Omega R_c)(\cos\beta z - \cos\beta h/\cos\beta h). \quad (51)$$

For the receiving antenna this expression corresponds to (53) in reference 1 for an extremely thin driven antenna. The two formulas are quite different but actually reduce to the same trigonometric form whenever βh is an odd multiple of $\pi/2$. After defining the current I_0 at the center of the receiving antenna by setting $z=0$ in (51) one obtains forms for the receiving antenna corresponding to (55) and (56) in reference 1 for the driven antenna. They are

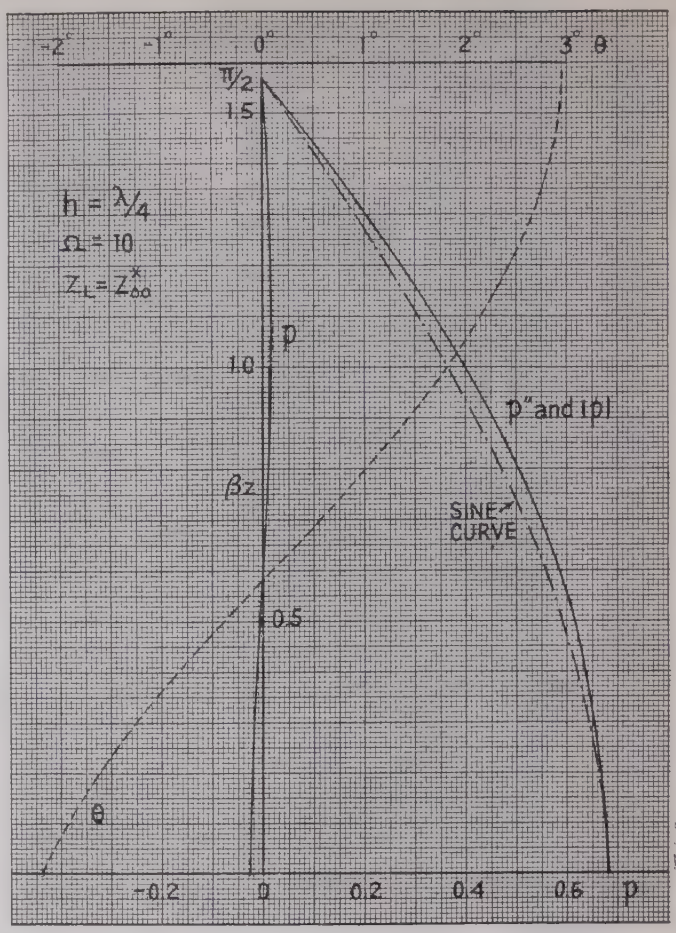


Fig. 12—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=\lambda/4$, $\Omega=10$. p'' is proportional to the component of current in phase with the electric field, p' is proportional to the component in phase quadrature with the field; $|p|$ is proportional to the magnitude of the current; θ is its angle referred to the electric field.

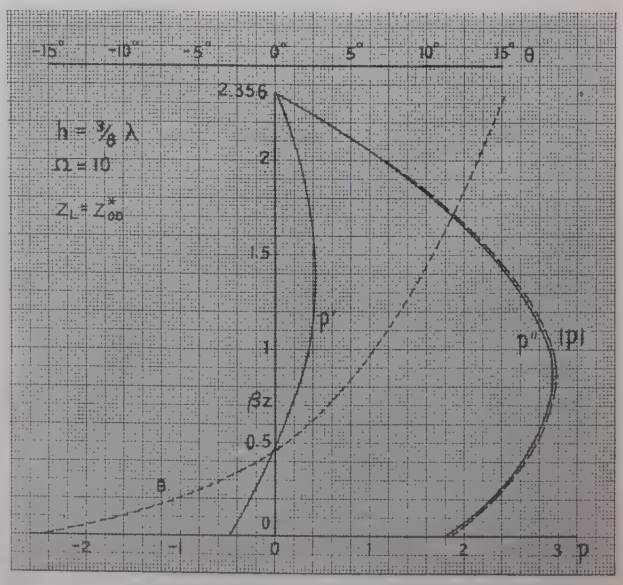


Fig. 13—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=3\lambda/8$; $\Omega=10$.

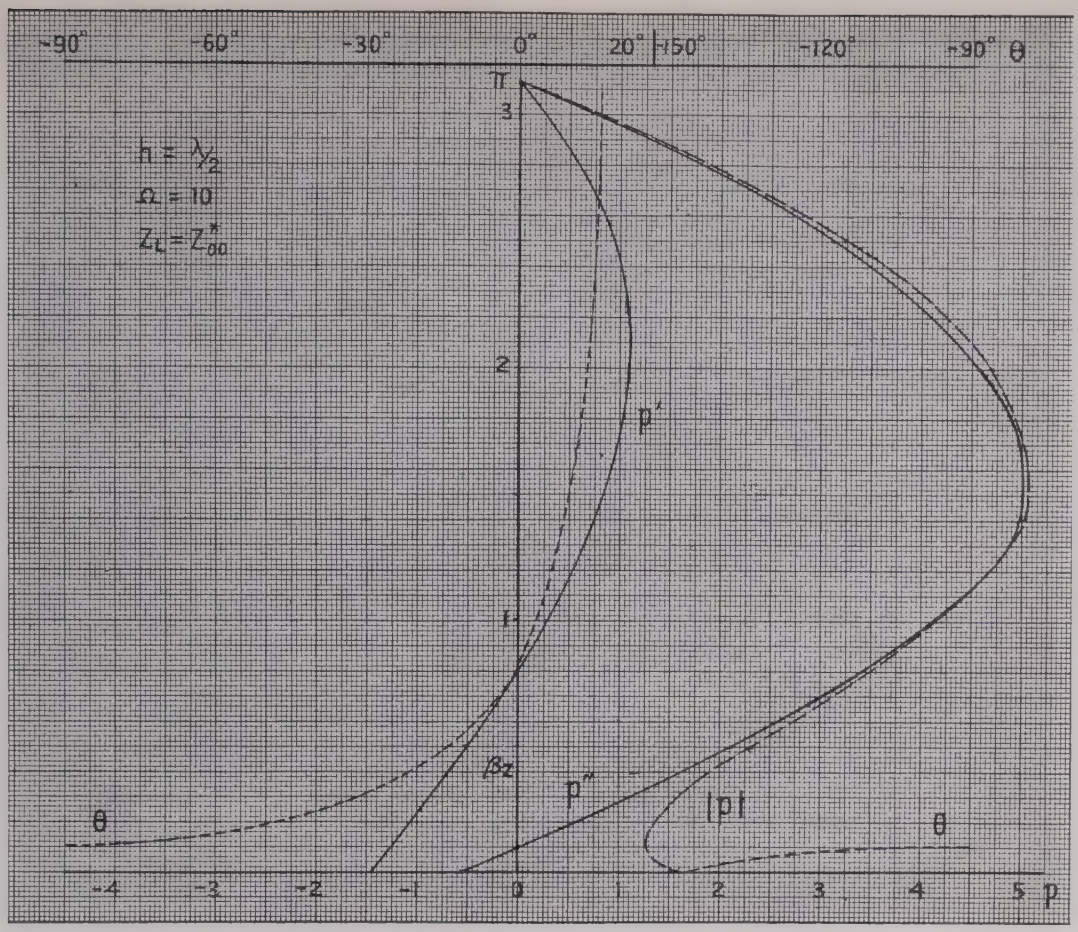


Fig. 14—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=\lambda/2$, $\Omega=10$.

$$I_0 = (j4\pi U/\Omega R_c)(1 - \cos \beta h/\cos \beta h) \quad (52)$$

$$I_z = I_0(\cos \beta_z - \cos \beta h/1 - \cos \beta h). \quad (53)$$

The formula (51) is, of course, a good approximation for thicker antennas over the limited ranges of the variables βz and βh for which the trigonometric terms in numerator and denominator of (35) are large compared with the terms in $1/\Omega$. When βh approaches an odd multiple of $\pi/2$, $\cos \beta h$ vanishes. Near such values one can write

$$I_0 = (j4\pi U/\Omega R_c)\{(1 - \cos \beta h)/(\cos \beta h + (1/\Omega)(A_1^I + A_1^{II}))\} \quad (54)$$

with (53) unchanged. Because the component of current in phase with U is always relatively small compared with the quadrature component except with h near odd multiples of $\lambda/4$ when the distributions of both components are alike, the approximation (53) is a much better one for a receiving antenna than is (56) of reference 1 for a driven antenna. Moreover, one is usually interested in the current at the center of the antenna, and (53) is actually a good approximation for I_z for all values of βh sufficiently below 2π. With $\beta h = 2\pi$, I_z apparently becomes infinite. Actually I_0 simultaneously vanishes and I_z remains finite. Since receiving antennas that satisfy (29) and which have βh much greater than 4 are of no practical importance even when loaded at the center (as will be shown below) (53) with (52), or if required with (54), is a reasonably satisfactory approximation for many purposes.

The Distribution of Current Along a Center-Loaded Receiving Antenna

In normal operation a receiving antenna is connected to a load impedance (which usually consists of a network of tuned circuits) either directly or by means of a transmission line. In some instances it is necessary or desirable to design the load and the antenna so that a maximum of power is transferred from the distant transmitter to the load. In other cases the receiver itself is made sufficiently sensitive so that it need abstract only a negligible amount of power and practically all the power received by the antenna is reradiated. This case then reduces approximately to that of an unloaded antenna.

Consider the general case illustrated in Fig. 11 of a receiving antenna placed in a plane perpendicular to the line joining transmitter and receiver. A load impedance Z_L (which may be the input impedance of a transmission line that is loaded at the output end) is connected across two terminals at its center. These are assumed to be sufficiently close together so that they are in the near zone with respect to each other. With the aid of the compensation theorem the analysis of this circuit can be reduced mathematically to that of an equivalent circuit

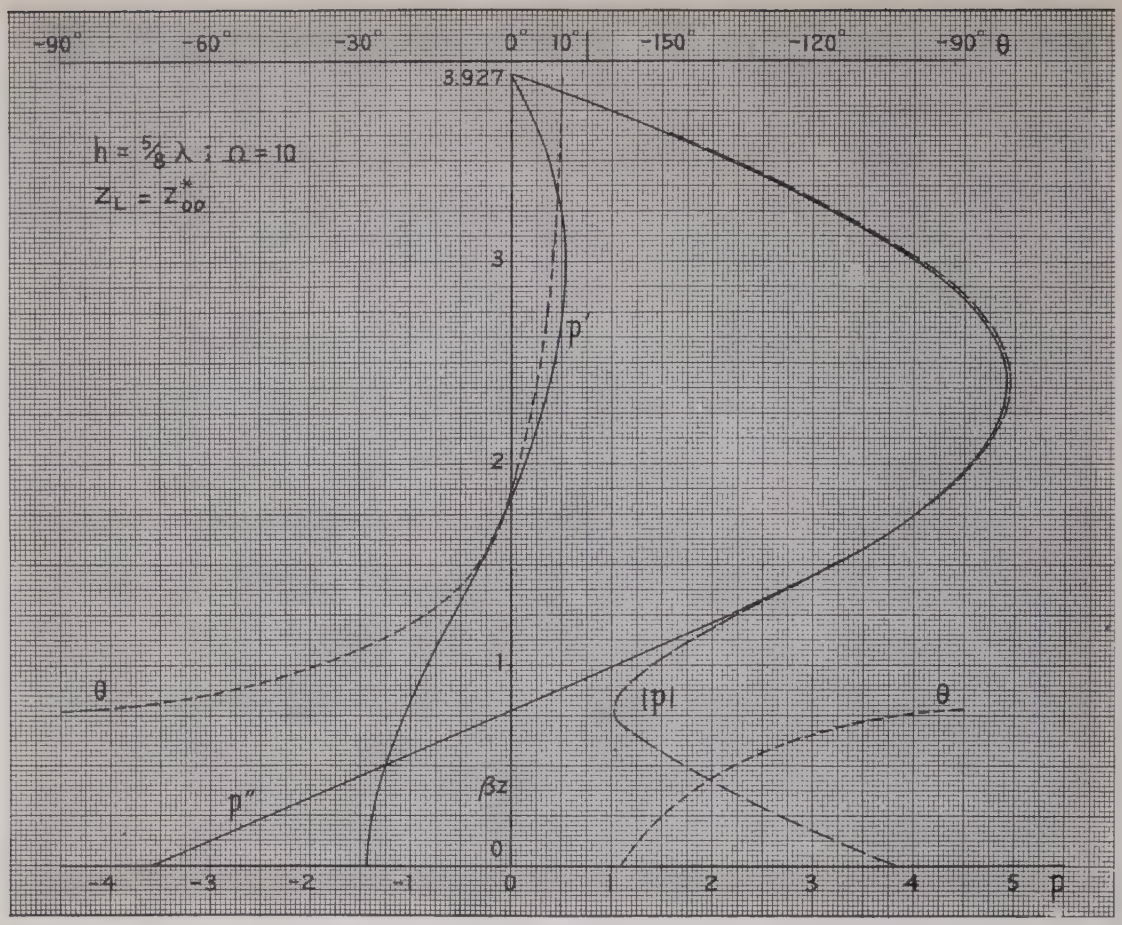


Fig. 15—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=5\lambda/8$, $\Omega=10$.

consisting of an unloaded receiving antenna with an impedanceless generator at its center. According to this theorem any impedance Z_L which carries a current I_0 may be replaced by an impedanceless generator with an impressed potential difference or electromotive force $V_L{}^e$ equal to the potential drop across the impedance. That is, with

$$V_L{}^e = I_0 Z_L. \tag{55}$$

If this substitution is made the circuit on the right in Fig. 11 is equivalent to that on the left.

By direct application of the principle of superposition the total current I_0 is the algebraic sum of the current I_{0V} due to the generator, and the current I_{0E} due to the action of the distant transmitter as expressed in the electric field E thus,

$$I_0 = I_{0E} - I_{0V}. \tag{56}$$

The current I_{0V}, due to the impedanceless generator, is the same as that in a driven antenna. Thus,

$$I_{0V} = (V_L{}^e/Z_{00}) = I_0(Z_L/Z_{00}). \tag{57}$$

Here Z_{00} is the input self-impedance of a center-driven antenna which has the same dimensions as the receiving antenna under consideration.[4] With (55) and (57), (56) may be written as follows:

$$\begin{aligned} I_{0E} &= I_0 + I_{0V} = V_L{}^e((1/Z_L) + (1/Z_{00})) \\ &= I_0(1 + (Z_L/Z_{00})). \end{aligned} \tag{58}$$

At any point z along the antenna the current I_z is by the principle of superposition also the algebraic sum of the separate currents due to the electric field and due to the generator. Thus,

$$I_z = I_{zE} - I_{zV}. \tag{59}$$

With (46) and (43) in reference 1 this may be expanded as follows:

$$\begin{aligned} I_z = (1/30\Omega D)\{ & U[g''(z) + jg'(z)] \\ & - \tfrac{1}{2}V_L{}^e[f''(z) + jf'(z)]\}. \end{aligned} \tag{60}$$

With (58) and setting $z=0$ in (46), (55) becomes

$$\begin{aligned} V_L{}^e &= I_{0E}(Z_{00}Z_L/Z_{00} + Z_L) \\ &= (U/30\Omega D)[g''(0) + jg'(0)](Z_{00}Z_L/Z_{00} + Z_L). \end{aligned} \tag{61}$$

Upon substituting (61) in (60) one obtains the following general equation for the current in the loaded receiving antenna in terms of the real factors p'' and p' which are, respectively, proportional to the components of current in phase and in phase quadrature with U.

$$I_z = (U/30\Omega D)(p'' + jp'). \tag{62}$$

$$\text{Here } U = (E_{\theta_2}\cos\psi)/\beta = (\lambda E_{\theta_2}\cos\psi)/2\pi \tag{63}$$

$$\begin{aligned} p'' + jp' = & [g''(z) + jg'(z)] - (1/60\Omega D) \\ & (Z_{00}Z_L/Z_{00} + Z_L)(g''(0) + jg'(0)) \cdot [f''(z) + jf'(z)]. \end{aligned} \tag{64a}$$

$$\text{The notation, } |p| = \sqrt{(p')^2 + (p'')^2}; \qquad \theta = \tan^{-1}(p'/p'') \tag{64b}$$

[4] Ronold King and F. G. Blake, Jr., "The self-impedance of a symmetrical antenna," Proc. I.R.E., vol. 30, pp. 335–349; July, 1942.

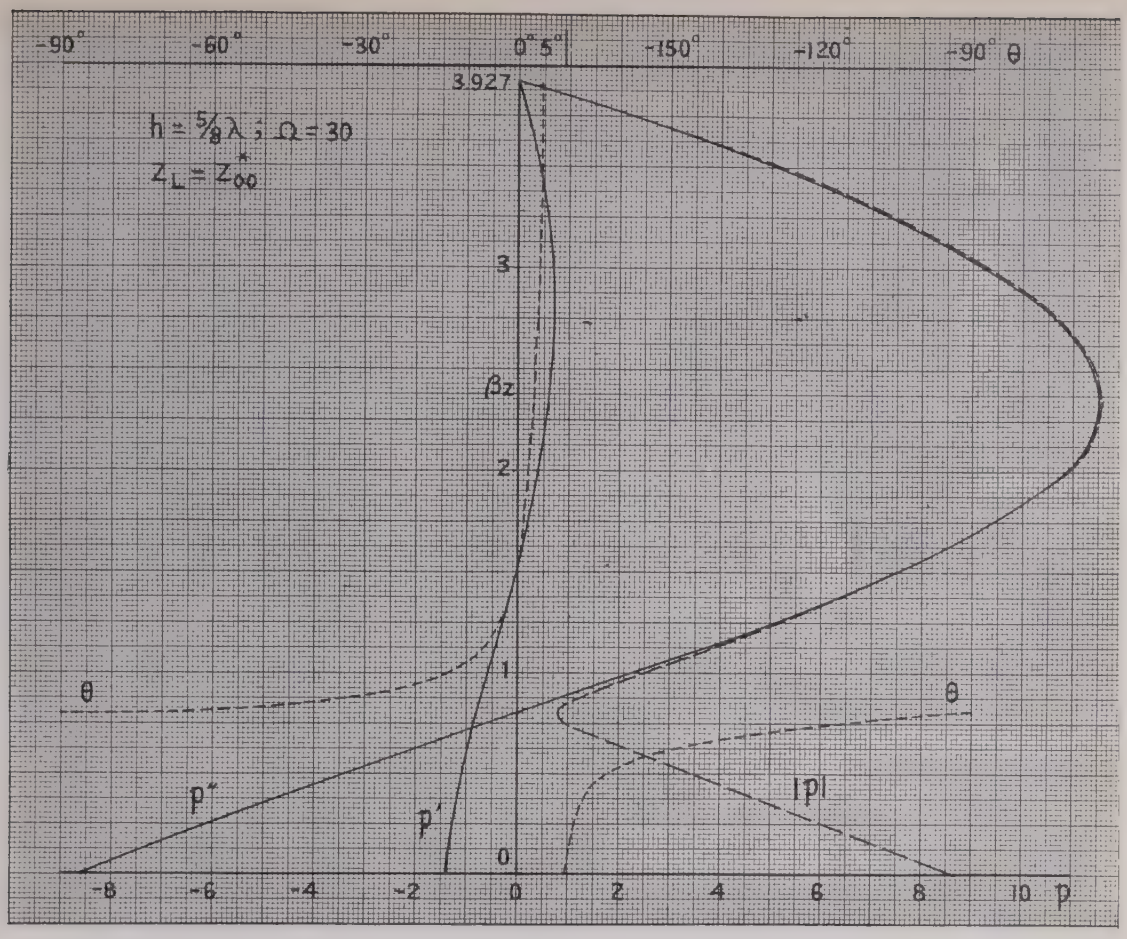


Fig. 16—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=5\lambda/8$, $\Omega=30$.

will be used. Since Z_{00} contains 60Ω as a factor, the second set of terms in (64a) is not negligible unless Z_L becomes sufficiently small. If the antenna is sufficiently thin (62) with (64a) becomes

$$I_z=(U/30\Omega\cos\beta h)\{j(\cos\beta z-\cos\beta h)+(1/60\Omega\cos\beta h)(Z_{00}Z_L/Z_{00}+Z_L)\cdot(1-\cos\beta h)\sin\beta(h-|z|)\}. \quad (65)$$

The distributions of current along a center-loaded receiving antenna for several different loads, lengths, and thicknesses are shown in Figs. 12 to 18 in terms of the functions p', p'', $|p|$, and θ. They were calculated from the general formula (62) using the distribution functions g' and g'' given in Figs. 3 to 6 and the distribution functions f' and f'' shown in reference 1, Figs. 7 to 10.

If the load is conjugate-matched to the antenna so that $Z_L=Z_{00}^*$ one has

$$(Z_{00}Z_L/Z_{00}+Z_L)=(Z_{00}Z_{00}^*/Z_{00}+Z_{00}^*)=(R_{00}^2+X_{00}^2/2R_{00})=(1/2G_{00}). \quad (66)$$

Figs. 12 to 16 were calculated using (66) in (64a). In Fig. 17 the load consisted of a reactance such that $X_L=-X_{00}$ while the resistance was made very much greater than the self-resistance R_{00} of the antenna. In Fig. 18, $X_L=-X_{00}$ while $R_L=0$. Very considerable differences in the distribution of current are seen to exist in antennas of the same length but with different impedances at the center. In particular, whenever the antenna and its load are made self-resonant the distribution of current resembles that along a center-driven antenna of the same length much more than that along an unloaded receiving antenna. This effect is increased if the load resistance is made as small as possible, as can be seen very readily from (64a). Clearly if the factor $(Z_{00}Z_L/Z_{00}+Z_L)(g''(0)+jg'(0))$ in (64a) is made sufficiently large and predominantly real, the distribution function $f''(z)+jf'(z)$ may be made to predominate. This is precisely the distribution function for a center-driven antenna. It may be approximated by requiring $R_L=0$; $X_L=-X_{00}$, i.e., by tuning the antenna to resonance with a pure reactance connected across its terminals. The distribution of current for these conditions is shown in Fig. 17 for one length and radius. The importance of this fact for coupled antennas will be considered in a subsequent paper.

Since the functions g' and g'', and p' and p'' are plotted in Figs. 3 to 18, a direct comparison of the amplitude of current for antennas of different thickness is possible only if these functions are multiplied by the factor $U/30\Omega D=\lambda E_{\theta_i}\cos\psi/60\pi\Omega D$. If the antennas (the currents in which are to be compared) are similarly oriented in the same electric field at the same frequency

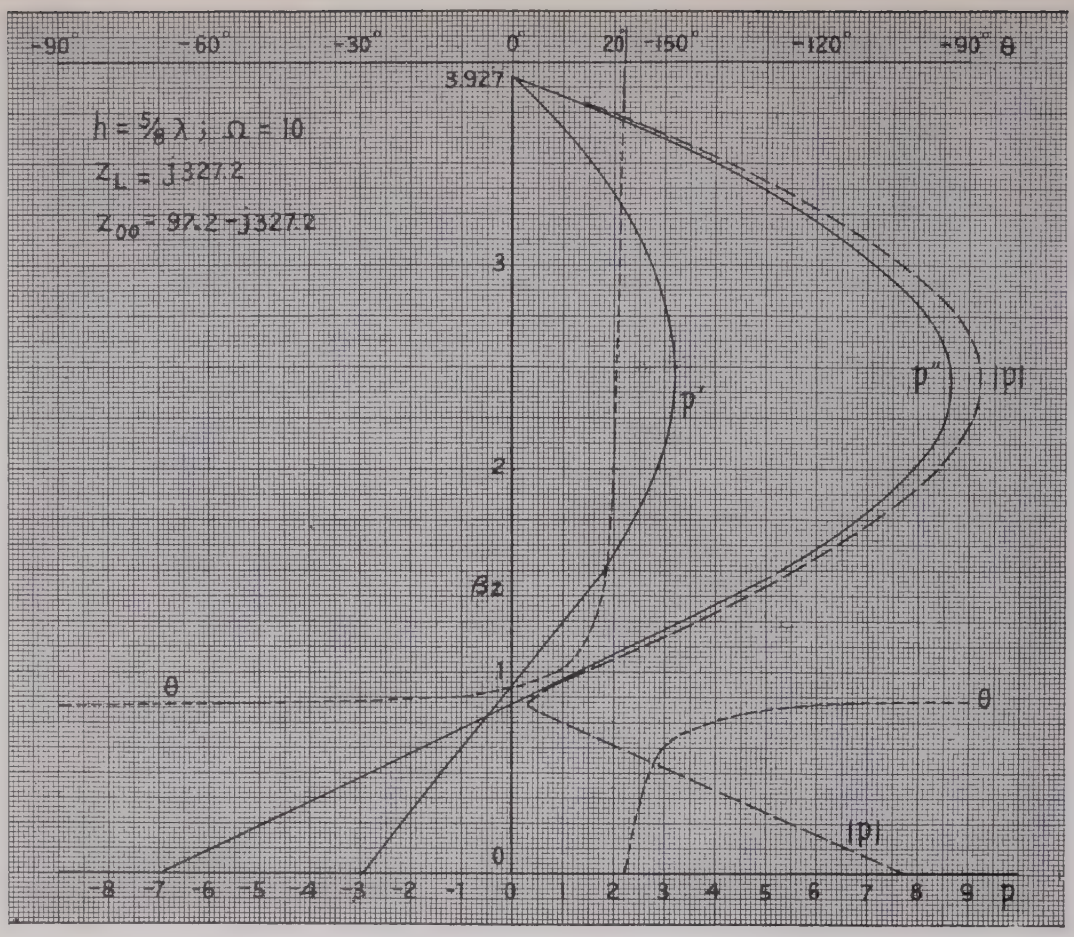


Fig. 17—The functions p', p'', $|p|$, and θ for a receiving antenna with a purely reactive load which is the negative of the input reactance of the antenna. $h=5\lambda/8$, $\Omega=10$.

multiplication by the factor $1/60\pi\Omega D$ is sufficient. This factor is given in Table I for antennas of two thicknesses and four lengths. In Table II are shown the

TABLE II

h/λ	$\Omega=2\ln\frac{2h}{a}$	$\lvert I_0\rvert/\lambda E_{\theta_1}\cos\psi$
0.25	10	9.35×10^{-3}
	30	8.10
0.375	10	3.08
	30	0.92
0.5	10	2.92
	30	0.82
0.625	10	3.91
	30	0.98

comparable magnitudes of the currents at the centers of unloaded receiving antennas of different lengths in the form $|I_0|/\lambda E_{\theta_2}\cos\psi$. It is clear that the amplitude of current is large near self-resonance and small near antiresonance. Precisely self-resonant and antiresonant lengths are not tabulated. In Fig. 19 the magnitude $|I_z/\lambda E_{\theta_2}\cos\psi|$, is plotted for all antennas of half-length $h=(5/8)\lambda$ previously computed and separately plotted in order to make ready intercomparison of both magnitudes and distributions of current possible under different load conditions and for different thicknesses but when immersed in exactly the same field.

The Equivalent Circuit and the Effective Length of a Loaded Receiving Antenna Oriented to Lie in an Equiphase Plane of a Linearly Polarized Electric Field

The relation (58) may be rearranged as follows:

$$I_{0E}Z_{00} = I_0(Z_{00}+Z_L). \tag{67}$$

This equation gives the correct amplitudes of the current I_0 entering or leaving the load Z_L connected at the center of a symmetrical receiving antenna that is placed in the plane perpendicular to the line joining it to a distant transmitter. Actually, however, it is also the equation of the same antenna with the same load at its center but driven by an impedanceless generator as shown in Fig. 20. The electromotive force of the generator is

$$V_0^e = I_{0E}Z_{00}. \tag{68}$$

Thus the forces acting on the charges distributed all along the receiving antenna in the actual circuit are replaced in the "equivalent" circuit by forces due to a generator acting as a concentrated electromotive force at the center. For the purpose of determining the current I_0 the two circuits are equivalent; but not for all purposes. For example, the distribution of current along the receiving antenna is given by (62), whereas the distribution along the antenna in the equivalent circuit is

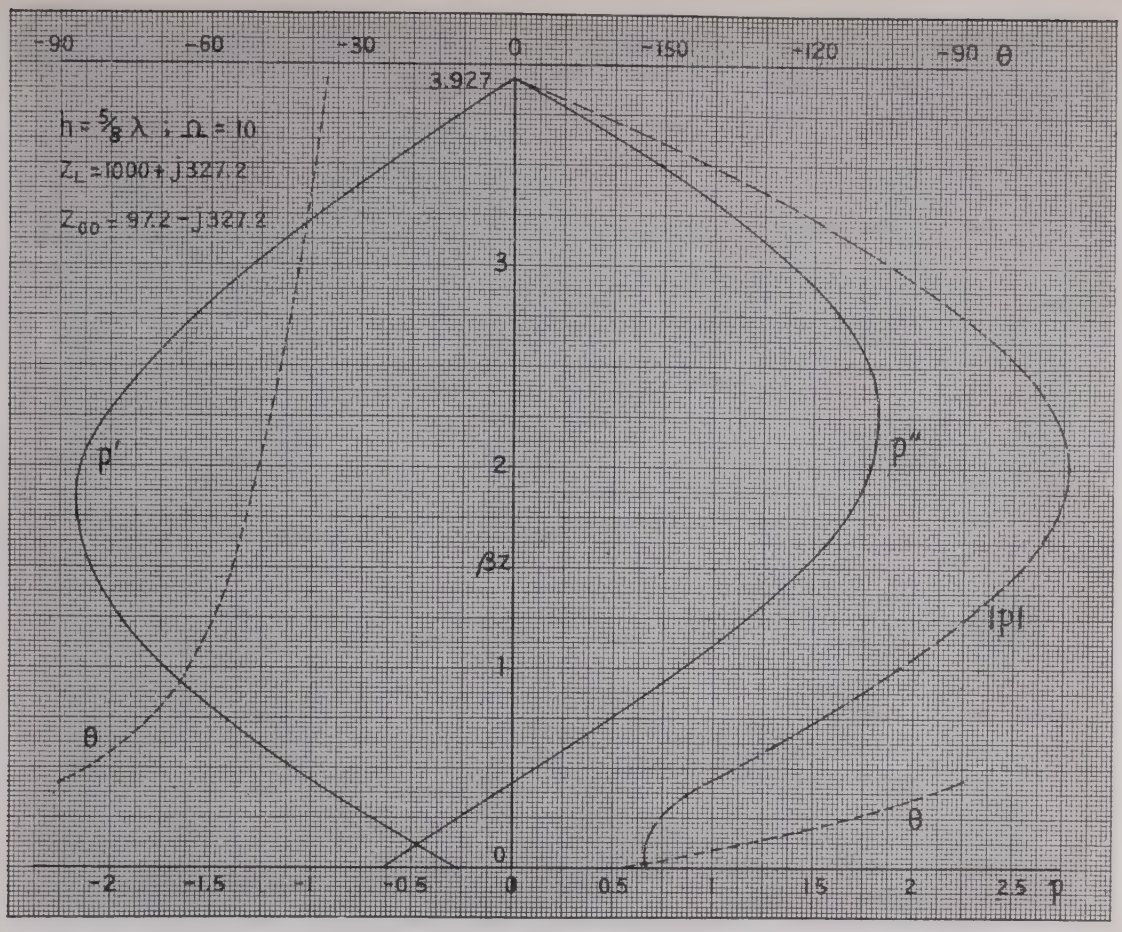


Fig. 18—The functions p', p'', $|p|$, and θ for a receiving antenna with a load including a resistance of 1000 ohms and a reactance which is the negative of the input reactance of the antenna. $h=5\lambda/8$, $\Omega=10$.

given by (43) of reference 1. The two distributions are not at all similar and they lead to quite different electromagnetic fields. That is, the power which would be radiated from the "equivalent" driven antenna would be transferred to different parts of the distant universe than is the power reradiated from the actual antenna. Whereas the fictitious electromotive force (68) is so chosen that the current I_0 is the same in the actual and the "equivalent" circuits, the total power radiated from the driven antenna in the "equivalent" circuit is not equal to the power actually reradiated from the receiving antenna. From the point of view of the power absorbed in the load Z_L, this is immaterial and the "equivalence" of the two circuits is entirely adequate.

The electromotive force V_0^e of the impedanceless generator in the "equivalent" circuit as defined by (68) is readily expressed analytically. Thus, with $z=0$ in (46),

$$V_0^e = (U/30\Omega D)\,[g''(0) + jg'(0)]Z_{00}. \tag{69}$$

It may be expressed directly in terms of the electric field using (29). Thus, defining a complex "effective" length, $2(h_e''+jh_e')=2h_e^{j\phi}$, for the antenna of actual length $2h$ one can write

$$V_0^e = 2\,|\,h_e\,|\,E_{\theta_2}\cos\psi\cdot e^{j\phi} \tag{70}$$

with $$h_e''+jh_e' = |\,h_e\,|\,e^{j\phi} = \frac{[g''(0)+jg'(0)]Z_{00}}{60\beta\Omega D}. \tag{71}$$

Here Z_{00} is the self-impedance of the antenna as seen by a generator connected in place of the load. In most cases involving a linearly polarized electric field one is not at all interested in the relative phase relations between the electromotive force in the "equivalent" series circuit and the electric field parallel to the antenna. Accordingly the angle ϕ is of little or no consequence and it is adequate to compute as the real "effective" half-length of the antenna the magnitude $|\,h_e\,|$ of (71). It is

$$|\,h_e\,| = (|\,g(0)\,|\,|\,Z_{00}\,|/60\beta\Omega D). \tag{72a}$$

The "effective" electrical length is

$$H_e = \beta h_e = (2\pi h_e/\lambda). \tag{72b}$$

The electromotive force in the equivalent series circuit then has the magnitude

$$|\,V_0^e\,| = 2\,|\,h_e\,|\,|\,E_{\theta_2}\,|\cos\psi. \tag{73}$$

It is easily verified by the application of Thévenin's theorem at the terminals of the antenna to which the load is attached that (70) must be the open-circuit voltage across these terminals. The effective length of a center-loaded, isolated antenna immersed in a plane-polarized electric field may be defined in general to be the open-circuit voltage at the load terminals of the antenna divided by the component of the electric field in the plane containing the antenna and the wave normal (a line perpendicular to a wave front or surface of

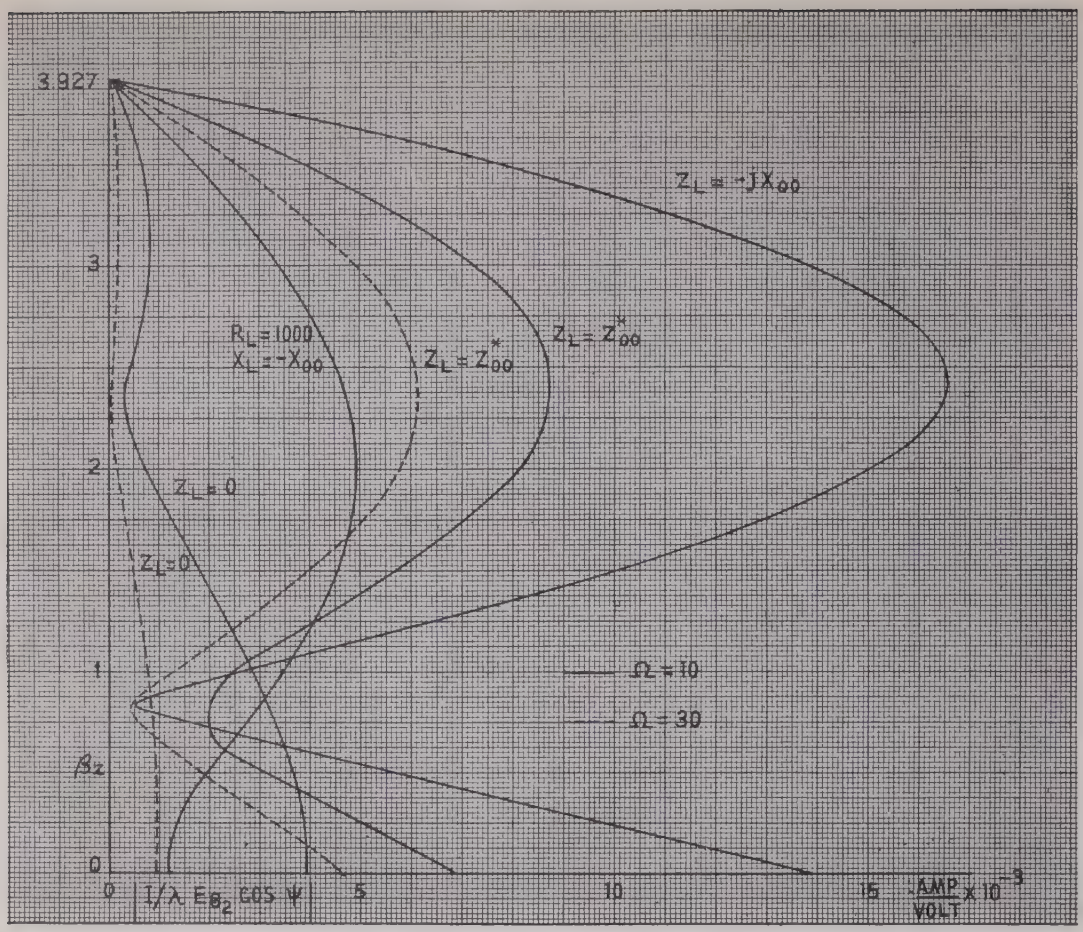


Fig. 19—The amplitudes $|(I/\lambda E_{\theta_2} \cos \psi)|$ for the antennas of Figs. 10 and 15 to 18. This figure permits comparison of the actual amplitudes of current in antennas of the same length immersed in the same electric field at the same frequency. $h = 5\lambda/8$.

constant phase). This component is $E_{\theta_2} \cos \psi$ in the notation of the present paper. It is the component parallel to the antenna *only in the special case defined by* (28), i.e., when the antenna is itself perpendicular to the wave normal so that it lies in a plane of constant phase. For an indefinitely thin antenna using (54) of reference 1 this reduces to the real form given below.[5,6]

$$h_e = (1 - \cos \beta h/\beta \sin \beta h) = (\lambda/2\pi) \tan (\tfrac{1}{2}\beta h). \quad (74)$$

[5] The effective half-length or height of a symmetrical antenna (or the effective full length or height of a vertical antenna erected over a perfect conductor) has been defined to be $\int I_z dz$ divided by I_0, where I_z is the total vertical current in the antenna structure at height z and I_0 is the current at the point where power is transferred to or from the connected apparatus. This definition of "effective height" for a receiving antenna leads approximately to the simple special form (74) for an indefinitely thin antenna only in the special case of a load for which $X_L = -X_{00}$. Only in this case is the simple distribution function $I_z = I_0 \sin \beta(h - |z|)$ a moderately good approximation in a receiving antenna, and it is this function which, when substituted in $(\int I_z dz)/I_0$, gives (74). It has already been pointed out[6] that it is possible to define an effective height for a receiving antenna in terms of $(\int I_z dz)/I_0$ using the correct distribution of current for an unloaded receiving antenna. But the effective height so defined does not, when multiplied by the component of the electric field parallel to the antenna, give the electromotive force of a fictitious generator in series with a simple circuit consisting of the load and the input impedance of the antenna as a driven antenna, as does that defined by (71) (or approximately over a limited range that defined by (74) in the special case defined by (28).

[6] Ronold King, "The approximate representation of the distant field of linear radiators," Proc. I.R.E., vol. 29, p. 461; August, 1941.

Formulas (72a) and (74) are plotted in Fig. 21. It is important to note that the simple form (74) is a good approximation even for thick antennas for lengths sufficiently below $h = \lambda/2$. The maxima of (72a) occur at the antiresonant lengths as obtained from (29b) of reference 4. Using the "effective" half-length given by (72) or

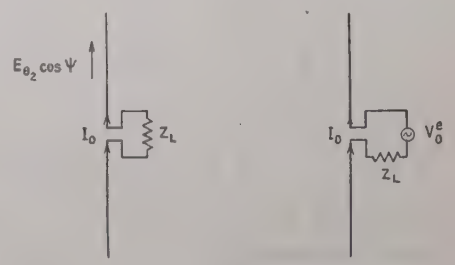


Fig. 20—"Equivalent" circuit of a loaded receiving antenna for determining current in the load.

where the approximation is adequate, by (74), the electromotive force in the "equivalent" series circuit of the receiving antenna is readily determined from (70) or (73) if the component of the electric field parallel to the antenna is known either from computation or by measurement and the antenna lies in an equiphase surface. It is important to point out again that the above

relations are valid *only if the receiving antenna is* (1) *in the radiation zone of the transmitter and* (2) *in the plane perpendicular to the line joining receiving and transmitting antennas* or more generally, perpendicular to the wave normal. The effective length of an antenna which satisfies (1) but not (2) is considered in a companion paper.[3]

Maximum Transfer of Power to the Load

The condition of maximum transfer of power to the load in the "equivalent" series circuit is that of a conjugate match as defined by $Z_L = Z_{00}^*$. In this case the total power supplied to the load is given by

$$P_L = (|V_0^e|^2/4R_{00}) = (|E_{\theta_2} h_e|^2 \cos^2\psi/R_{00}). \quad (75)$$

Here V_0^e and E_{θ_2} are root-mean-square values. With (72) this reduces to

$$P_L = (E_{\theta_2} \cos\psi/60\beta\Omega D)^2(|g(0)|^2/G_{00});$$
$$|g(0)| = \sqrt{[g'(0)]^2 + [g''(0)]^2}. \quad (76)$$

This may be written in the form

$$P_L = ((E_{\theta_2}\lambda \cos\psi)^2/240\pi^2) f(H) \quad (77a)$$

with

$$f(H) = (60|g(0)|/(60\Omega D)^2 G_{00}). \quad (77b)$$

For an indefinitely thin antenna, for which (74) may be substituted in (75), one can also write

$$R_{00} = R_m^e/\sin^2 H. \quad (78)$$

Here R_m^e is the radiation resistance referred to maximum sinusoidal current as plotted in Fig. 21[1] in the curve marked $\Omega = \infty$. The abbreviation $H = \beta h$ is used. With (74) and (78) used in (75),

$$P_L = (E_{\theta_2} \cos\psi/\beta)^2 \cdot ((1 - \cos H)^2/R_m^e). \quad (79)$$

This may be written in the alternative form

$$P_L = ((E_{\theta_2}\lambda \cos\psi)^2/240\pi^2) f(H) \quad (80a)$$

with

$$f(H) = (60(1 - \cos H)^2/R_m^e). \quad (80b)$$

This is equivalent to the result given by Hallén's,[2] formula (45) for the special case defined by (28). Curves

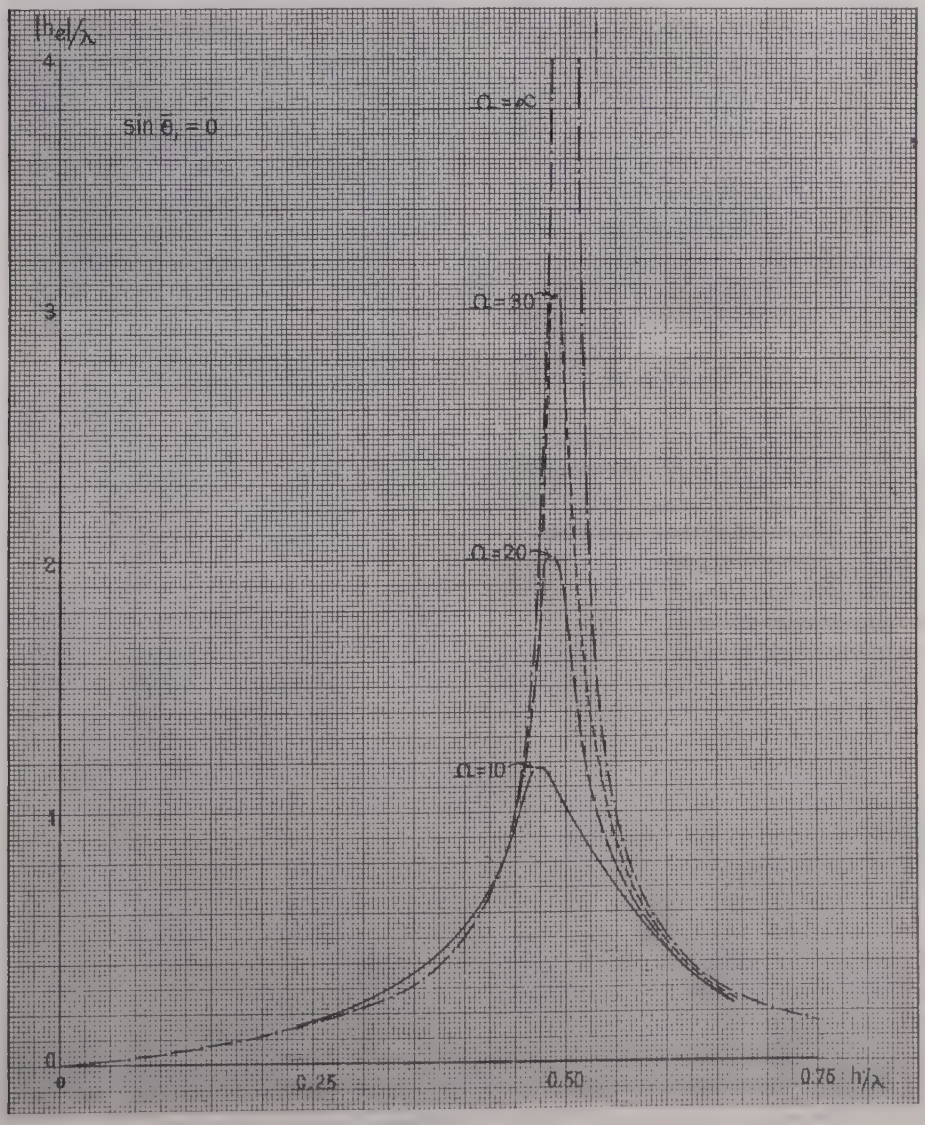


Fig. 21—The effective half-length $|h_e|$ of center-loaded cylindrical antennas of half-length h, or the effective length $|h_e|$ of base-loaded cylindrical antennas erected vertically over a perfectly conducting half space.

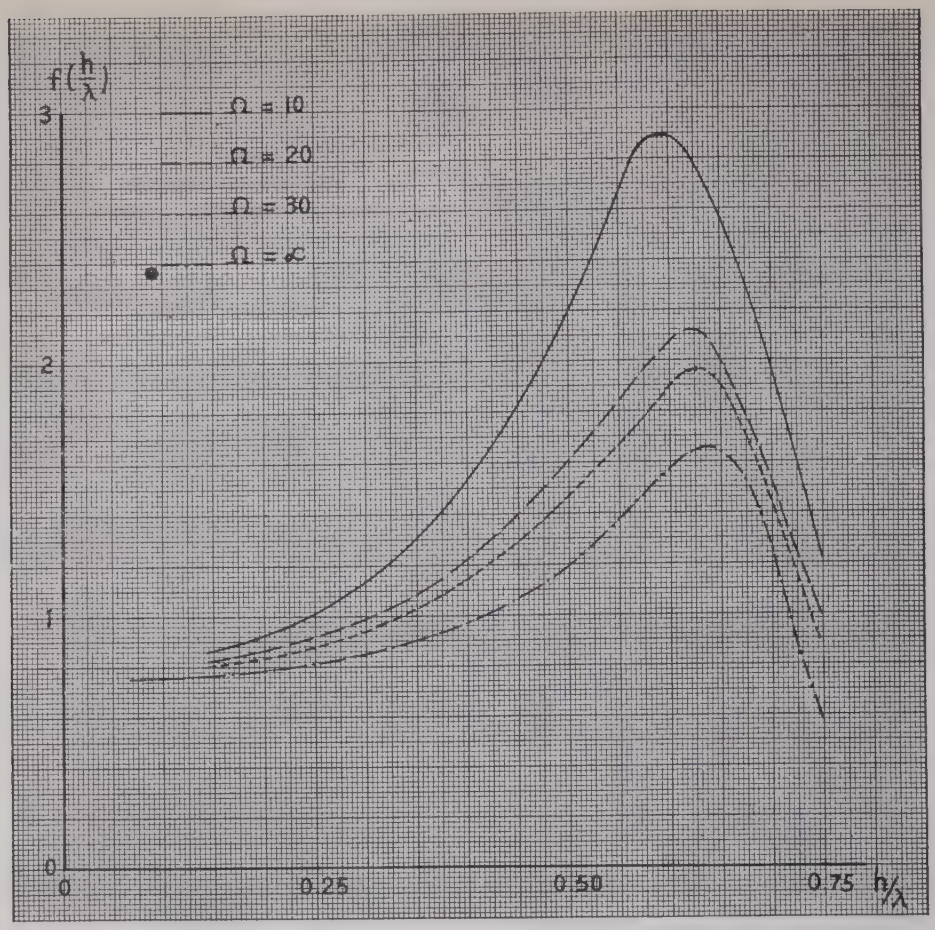


Fig. 22—The function $f(h/\lambda)$ which is proportional to the power transferred to a conjugate-matched load in a center-loaded receiving antenna.

of $f(H)$ as a function of H as computed from (77b) for three thicknesses as defined by $\Omega=10$, 20, 30, are shown in Fig. 22, together with the curve computed from (80b) and defined by $\Omega=\infty$. It is seen that the *optimum length for transferring a maximum power to a matched load is somewhat below the half length* $h=(3/8)\lambda$ *and more than twice the self-resonant half-length near* $h=\lambda/4$, *which gives maximum currents with no load.* Moreover the power transferred to the matched load is greater for thick than for thin antennas.

Appendix

$$
\begin{aligned}
S_1(z)=&-(\cos qz-\cos qh)\ln(1-z^2/h^2)\\
&+\tfrac{1}{2}\cos qz[\overline{Ci}(\beta+q)(h+z)+\overline{Ci}(\beta-q)(h-z)\\
&+\overline{Ci}(\beta-q)(h+z)+\overline{Ci}(\beta+q)(h-z)\\
&+jSi(\beta+q)(h+z)+jSi(\beta-q)(h-z)\\
&+jSi(\beta+q)(h-z)+jSi(\beta-q)(h+z)]\\
&-\tfrac{1}{2}\sin qz[Si(\beta+q)(h+z)+Si(\beta-q)(h-z)\\
&-Si(\beta-q)(h+z)-Si(\beta+q)(h-z)\\
&-j\overline{Ci}(\beta+q)(h+z)-j\overline{Ci}(\beta-q)(h-z)\\
&+j\overline{Ci}(\beta-q)(h+z)+j\overline{Ci}(\beta+q)(h-z)]\\
&-\cos qh[\overline{Ci}\beta(h+z)+\overline{Ci}\beta(h-z)\\
&+jSi\beta(h+z)+jSi\beta(h-z)]\\
&+\frac{j4\pi z^i h}{R_c}\left[\frac{\beta(\cos qz-\cos\beta z)}{h(\beta^2-q^2)}\right.\\
&\left.-\frac{\cos qh}{\beta h}(1-\cos\beta z)\right]. \qquad (81)
\end{aligned}
$$

$$S_1(z)=0 \text{ for } q=0 \qquad (82)$$

$$
\begin{aligned}
S_1(h)=&\tfrac{1}{2}\cos qh[\overline{Ci}2h(\beta+q)+\overline{Ci}2h(\beta-q)+jSi2h(\beta+q)\\
&+jSi2h(\beta-q)]-\tfrac{1}{2}\sin qh[Si2h(\beta+q)-Si2h(\beta-q)\\
&-j\overline{Ci}2h(\beta+q)+j\overline{Ci}2h(\beta-q)]\\
&-\cos qh[\overline{Ci}2\beta h+jSi2\beta h]\\
&+j\frac{4\pi z^i h}{R_c}\left[\frac{\beta(\cos qh-\cos\beta h)}{h(\beta^2-q^2)}\right.\\
&\left.-\frac{\cos qh}{\beta h}(1-\cos\beta h)\right]. \qquad (83)
\end{aligned}
$$

$$S_1(h)=0 \text{ for } q=0. \qquad (84)$$

$$Six=\int_0^x\frac{\sin u}{u}\,du \qquad (85)$$

$$\overline{Cix}=\int_0^x\frac{1-\cos u}{u}\,du \qquad (86)$$

z^i is the internal impedance per unit length of the antenna.

The Receiving Antenna in a Plane-Polarized Field of Arbitrary Orientation*

CHARLES W. HARRISON, JR.†, ASSOCIATE, I.R.E., AND RONOLD KING†, SENIOR MEMBER, I.R.E.

Summary—**The analysis previously made for a center-loaded linear receiving antenna of finite cross section oriented in the plane of a plane-polarized electric field is extended to include arbitrary orientation with respect to a linearly or elliptically polarized field. A general formula for the complex open-circuit voltage at the load terminals is given and this is expressed in terms of a complex effective length. The relation between the magnitude of this effective length and the field characteristic of the same antenna when driven is pointed out. By means of the reciprocal theorem the complex effective length is shown to yield a general expression for the radiation field of a center-driven antenna of nonvanishing radius. This is correlated with the corresponding expression previously obtained using an approximate simple representation of the distribution of current. Curves are shown for the two components and the magnitude of the complex effective length as functions of the length, the radius, and the orientation of the antenna in a linearly polarized electric field. Application is made to elliptically polarized fields which includes the case of ionospheric reflection. Numerical examples involving the computation of current in a loaded receiving antenna for several typical fields of different polarizations are given.**

INTRODUCTION

THE present paper is a sequel to an earlier investigation on the receiving antenna;[1] it also has an important bearing on the determination of the distant electric field of linear radiators of nonvanishing radius as analyzed in two recent papers.[2,3] Reference to numbered equations in any one of these three papers will be made by prefixing, respectively, 1-, 2-, or 3- to the appropriate equation number. Similarly the prefix 4- will be used for designating equations in the closely related article on the input impedance of an antenna.[4]

The analyses of the receiving antenna in reference 1 included a complete formulation of the general problem of two coupled antennas of nonvanishing radii. The detailed solution was, however, limited to the case of a receiving antenna which is sufficiently far from the transmitter that the electric field could be assumed to be approximately plane-polarized. The numerical solution in the form of curves and graphs was still further restricted to the practically most important case in which the receiving antenna was required to be in an equiphase plane of a linearly polarized electric field; a case illustrated by Fig. 1 if $\bar{\theta}_1 = 0$ and $\theta_1 = \pi/2$. In view of the fact that a mere superposition of two suitably oriented and phased linearly polarized electric fields yields an elliptically polarized field (which includes the special case of circular polarization) it follows that the special case treated in reference 1 actually includes the case of a receiving antenna *which lies in an equiphase plane* of a plane-polarized electric field, i.e., in a plane for which $\bar{\theta}_1 = 0$ or $\theta_1 = \pi/2$ in Fig. 1.

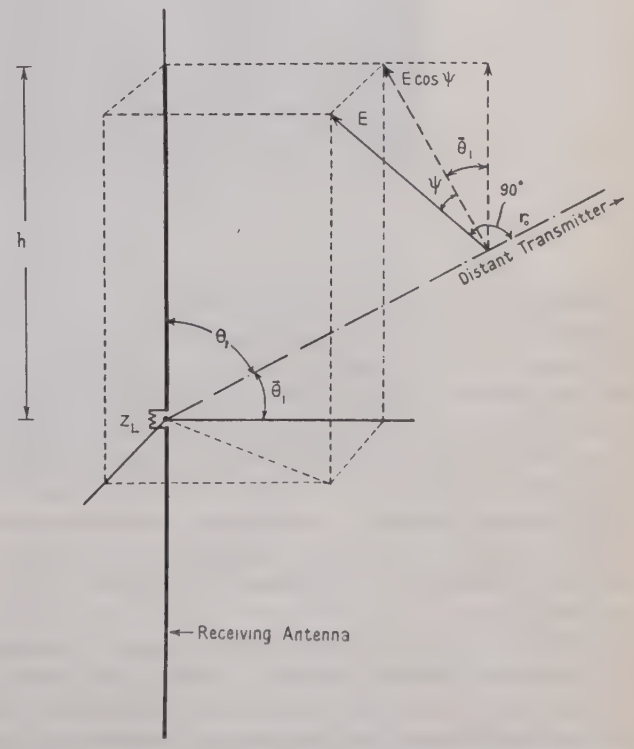


Fig. 1—Symmetrical, center-loaded receiving antenna in a linearly polarized electric field of arbitrary orientation. $E = -E_{\theta_2}$.

The report which is given below extends the numerical solution to include the much more general case of a receiving antenna which is oriented in an arbitrary way with respect to a linearly polarized electric field. That is, in Fig. 1 the angle $\bar{\theta}_1$ may have any value. By a

* Decimal classification: R120. Original manuscript received by the Institute, July 13, 1943; revised manuscript received, September 7, 1943.

† Cruft Laboratory and the Research Laboratory of Physics, Harvard University, Cambridge, Massachusetts.

[1] Ronold King and Charles W. Harrison, Jr., "The receiving antenna," PROC. I.R.E., this issue, pp. 18–35.

[2] Ronold King and Charles W. Harrison, Jr., "The distribution of current along a symmetrical center-driven antenna," PROC. I.R.E., vol. 31, pp. 548–567; October, 1943.

[3] Charles W. Harrison, Jr., and Ronold King, "The radiation field of a symmetrical center-driven antenna of finite cross section," PROC. I.R.E., vol. 31, pp. 693–698; December, 1943.

[4] Ronold King and F. G. Blake, Jr., "The self-impedance of a symmetrical antenna," PROC. I.R.E., vol. 30, pp. 335–349; July, 1942.

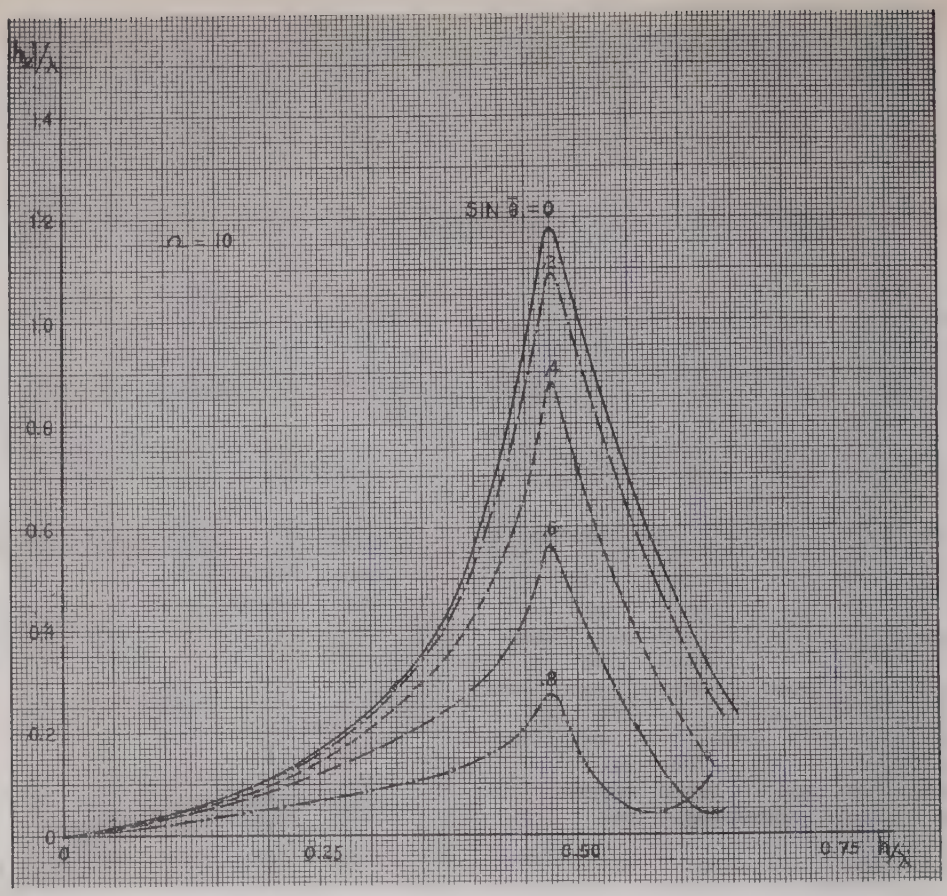


Fig. 2—$|h_e|/\lambda$ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and with $\Omega \equiv 2 \ln 2h/a = 10$.

combination of two such fields the case of an antenna oriented at will in an elliptically polarized electric field is obtained. This includes, for example, the highly important problem of a receiving antenna immersed in an electric field which is the resultant of one component due directly to the retarded action of currents in the transmitting antenna and of a second component due to the retarded action of periodically moving electrons in the ionosphere. This second component will, in general, differ from the first in phase, in amplitude, and in polarization.

Before proceeding to outline the analysis, it is perhaps well to point out that elementary treatments of the receiving antenna usually assume by implication and without justification the validity of the following three statements.

1) The distribution of current along a physically available receiving antenna is the same as that along an antenna of infinitely small radius.

2) The distribution of current along a receiving antenna, whether loaded or not, is the same as along a center-driven antenna of the same length.

3) The open-circuit voltage appearing across the load terminals of a receiving antenna which is immersed in an electric field due to a distant transmitter is proportional to the component of that field parallel to the axis of the antenna. In terms of Fig. 1 this means that the open-circuit voltage should be proportional to $E \cos \bar{\theta}_1 \cos \psi$.

It has been shown in detail[1] that the distribution of current along a center-loaded receiving antenna of nonvanishing radius differs very considerably from that of an infinitely thin antenna. Moreover, it varies greatly with the nature of the load. In every instance it differs from that of a center-driven antenna previously described.[2] A similarity between the distribution of current along driven and receiving antennas is to be observed only for the case of a receiving antenna which has an input reactance which is the conjugate of the load reactance, i.e., an antenna with a tuned load. Under all other conditions, especially in the case of no load or of only a small load, no similarity whatsoever was shown to exist between receiving and driven antennas. It follows that statements 1 and 2 are in general incorrect; that they are at best crude approximations for *extremely thin antennas with tuned loads*.

Statement 3 is correct for an antenna of any length only if the antenna lies in an equiphase plane of a linearly polarized electric field, that is, specifically under the conditions leading to the solution carried through in

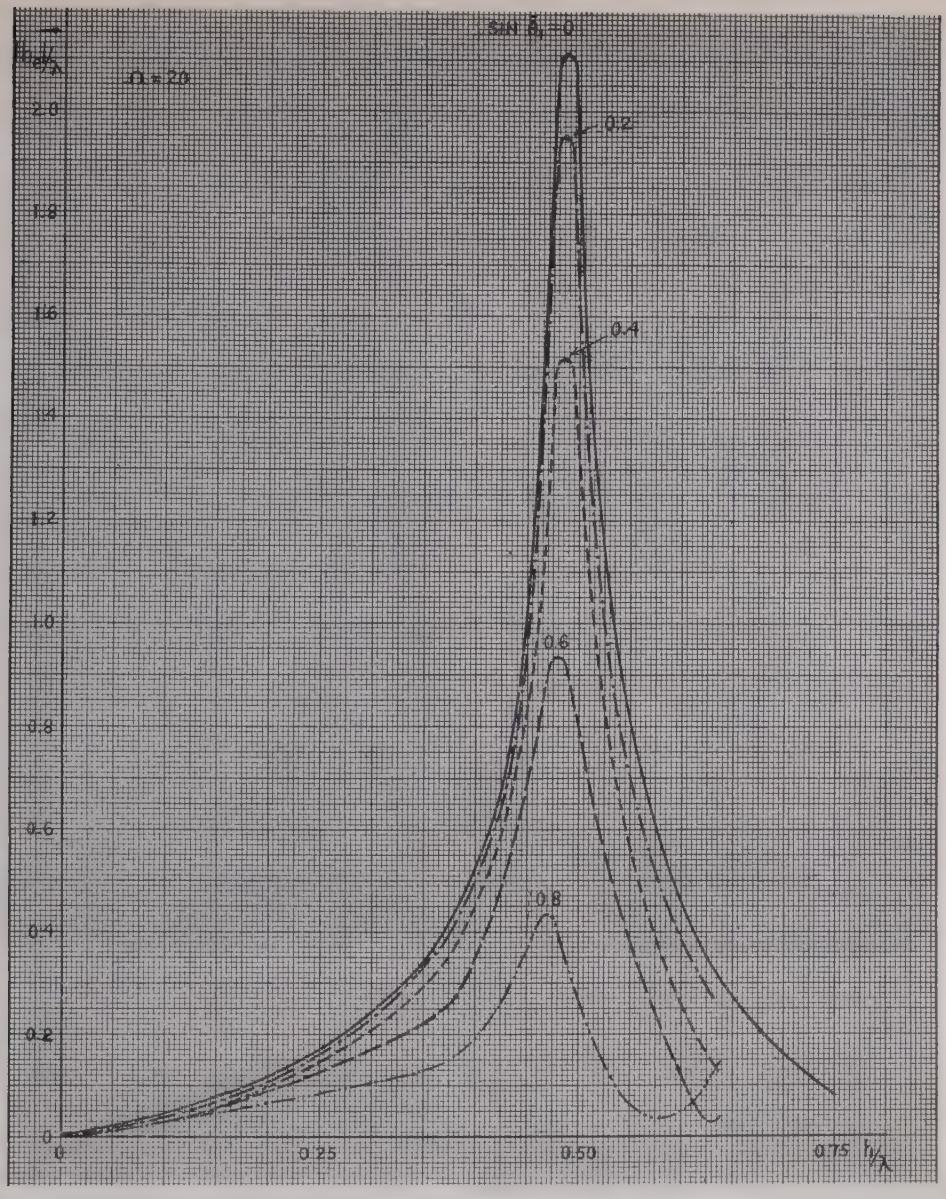


Fig. 3—$|h_e|/\lambda$ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and with $\Omega = 2 \ln 2h/a = 20$.

detail in reference 1, for which it was required that $\bar{\theta}_1 = 0$. In this case the open-circuit voltage was shown to be proportional to $E \cos \psi$. Since the antenna was required to be in the place of polarization which is also a surface of constant phase, all points along the antenna experienced an electric field *in the same phase* as well as in the same amplitude at any instant of time.

This can be visualized most simply by noting that all points along the receiving antenna are essentially at the same distance from the remote transmitter. If the antenna is not confined to an equiphase plane of polarization, i.e., if one end of the antenna may be inclined toward, the other away from, the distant transmitter, the amplitudes of the electric fields experienced by points near the two ends will still be sensibly the same, but not the phases. If one end is nearer the transmitter by an appreciable fraction of a wavelength than the other end the time delay will be significantly less. This means vastly different phase relations may obtain if the antenna is of considerable length and is appreciably inclined from the plane of constant phase. Hence, in this case, it is correct to assume statement 3 to be valid only if the receiving antenna is a very small fraction of a wavelength long. Since a rotation of the receiving antenna in a plane containing itself and a distant linear radiator is precisely the variation in position required to determine the vertical field pattern, the above conclusion is merely another way of stating that the vertical field pattern of a linear receiving antenna is of the form cos $\bar{\theta}_1$ or sin θ_1 only for a very short and extremely thin antenna. It will be recalled that for a center-loaded antenna of half-length h, and zero radius the "vertical" receiving pattern referred to the load current I_0, is

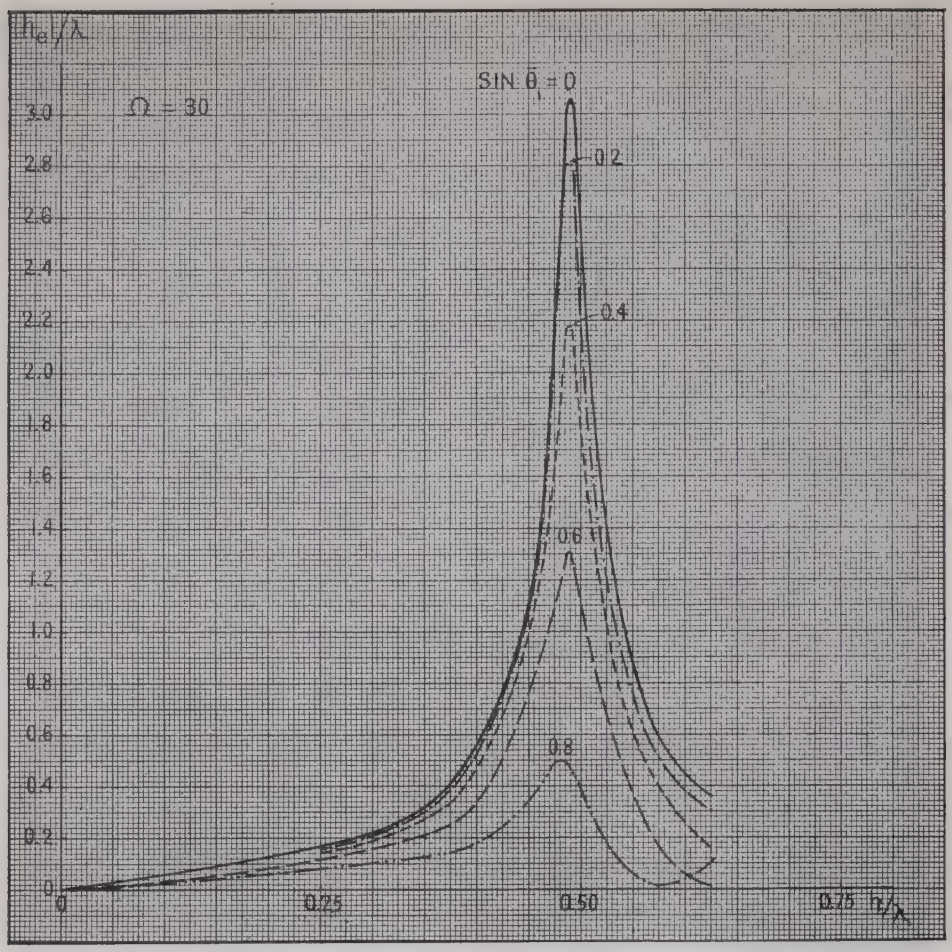


Fig. 4—$|h_e|/\lambda$ as a function of h/λ with $\sin \bar{\theta}_1$ as parameter and with $\Omega \equiv 2 \ln 2h/a = 30$.

$$V_0(\theta_1) = \frac{\cos(\beta h \cos\theta_1) - \cos\beta h}{\sin\beta h \sin\theta_1} \tag{1}$$

with $\beta = 2\pi/\lambda$. According to the reciprocal theorem, this is, of course, the same as the "vertical" field pattern for an identical, center-driven antenna. This will be considered in greater detail below. Thus, in general, it is not correct to write the open-circuit voltage of a receiving antenna of considerable length proportional to $E \cos\bar{\theta}_1 \cos\psi$. This proportionality is true for antennas of any length only for $\bar{\theta}_1 = 0$. It is, however, approximately true for all values of $\bar{\theta}_1$ for very short antennas.

The Effective Length of a Loaded Receiving Antenna Oriented in an Arbitrary Direction in a Linearly Polarized Electric Field

The formal definition of the complex effective length of a receiving antenna which is not required to be oriented in the plane of a linearly polarized electric field differs in no way from the definition for the special case previously described.[1] The final formula however is considerably more intricate due to the fact that more general functions are involved because the parameter,

$$q = \beta \sin\bar{\theta}_1, \tag{2}$$

cannot now be required to vanish. Thus, the current entering the load Z_L is obtained from (1-67) and (1-68). It is

$$I_0 = V_0^e/(Z_{00} + Z_L). \tag{3}$$

Here Z_{00} is, as before, the input self-impedance of the antenna[4] as if center-driven, and V_0^e is the driving potential of a fictitious generator of zero internal impedance connected in series with Z_{00} and Z_L. The function V_0^e is defined as before by (1-68), viz.,

$$V_0^e = I_{0E} Z_{00}. \tag{4}$$

Here I_{0E} is obtained from (1-35) by setting $z = 0$ but *without requiring q to vanish*. The resulting expression is

$$I_0 = \frac{j4\pi U}{\Omega R_c}\left\{\frac{[\cos qh - \cos\beta h] + 1/\Omega[m_1^I(0) + jm_1^{II}(0)]}{\cos\beta h + 1/\Omega[A_1^I + jA_1^{II}]}\right\}. \tag{5}$$

Here $R_c = 376.7$ and U in volts is defined by (1-27);

$$U = -(E_{\theta_2}\cos\psi/\beta\cos\bar{\theta}_1)_{z=0}. \tag{6}$$

Also,

$$\Omega = -2\ln(2h/a). \tag{7}$$

The functions, $[m_1^I(0) + jm_1^{II}(0)]$, are obtained from (1-37) with $z = 0$; $A_1^I + jA_1^{II}$ are the same as in (1-36).

The complex effective length $2h_e$ of the antenna of actual length $2h$ is defined as in (1-70) to satisfy the

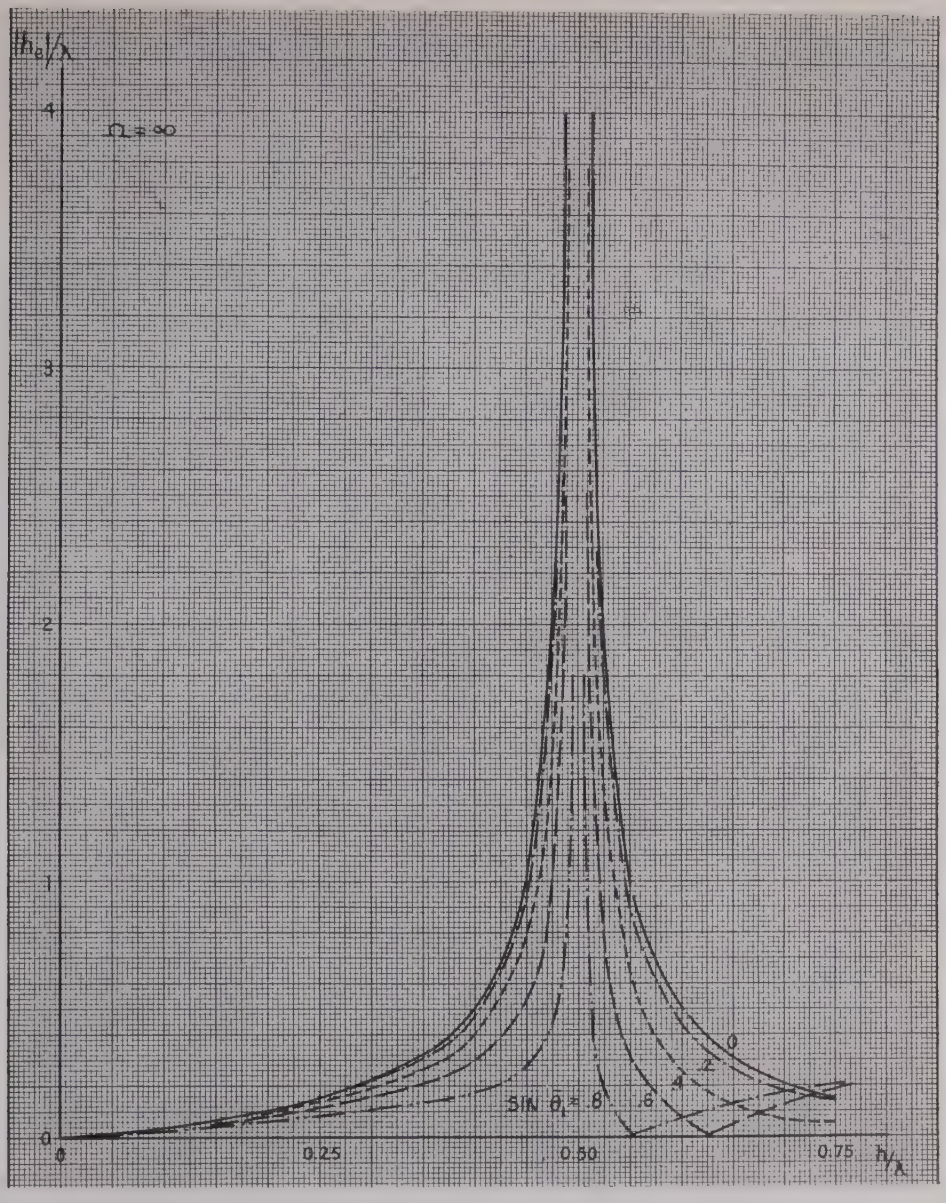


Fig. 5—h_e/λ as a function of h/λ with $\sin \bar{\theta}_1$ as parameter and with $\Omega = 2 \ln 2h/a = \infty$.

relation

$$V_0^e = 2h_e E_{\theta_2} \cos \psi. \tag{8}$$

Accordingly,

$$h_e = |h_e| e^{j\phi} = (h_e'' + jh_e') = V_0^e/(2E_{\theta_2} \cos \psi). \tag{9}$$

With (5) this yields the following expression for the effective half-length of a center-loaded antenna of length $2h$, or the effective length of a base-loaded antenna of length h when erected vertically over a perfectly conducing half space.

$$\frac{h_e}{\lambda} = \frac{jZ_{00}}{\Omega R_c \cos \bar{\theta}_1} \left\{ \frac{[\cos(\beta h \sin \bar{\theta}_1) - \cos \beta h] + 1/\Omega[m_1^I(0) + jm_1^{II}(0)]}{\cos \beta h + 1/\Omega[A_1^I + jA_1^{II}]} \right\}. \tag{10}$$

In the limiting case of an antenna of vanishingly small radius, i.e., as $a \to 0$ or $\Omega \to \infty$, one has from (2-54)

$$(Z_{00})_{a\to 0} = -j(R_c\Omega/2\pi) \cot \beta h \tag{11}$$

so that the effective half-length reduces to the following real form

$$\left(\frac{h_e}{\lambda}\right)_{a\to 0} = \frac{1}{2\pi} \left\{ \frac{\cos(\beta h \sin \bar{\theta}_1) - \cos \beta h}{\sin \beta h \cos \bar{\theta}_1} \right\}. \tag{12}$$

For a very short antenna which permits writing

$$(\beta h)^2 \ll 1 \tag{13}$$

(12) reduces to

$$(h_e)_{a\to 0} = \tfrac{1}{2} h \cos \bar{\theta}_1. \tag{14}$$

In this case, (8) reduces to

$$V_0^e = hE_{\theta_2} \cos \psi \cos \bar{\theta}_1; \; a \to 0 \tag{15}$$

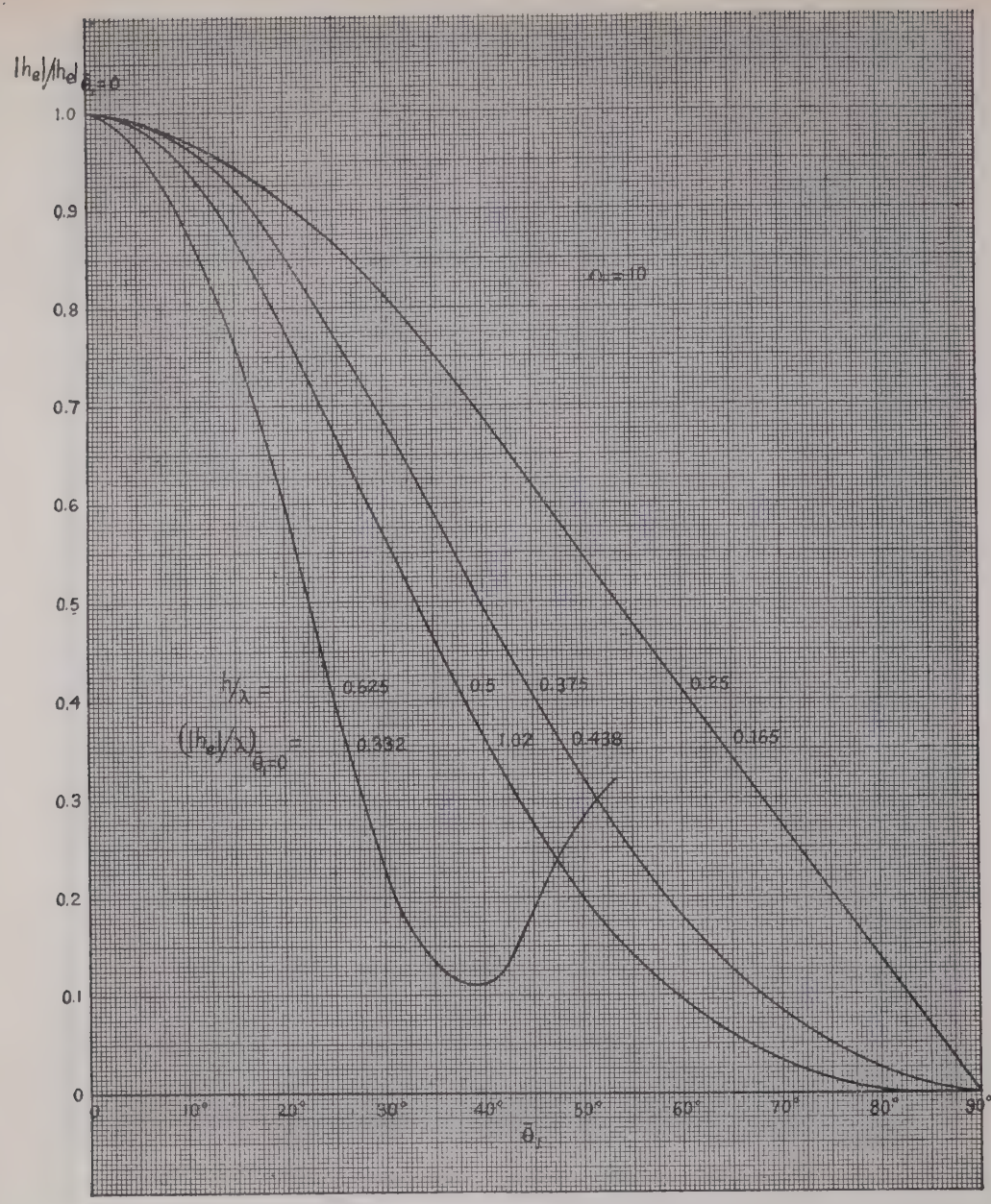


Fig. 6—$|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with h/λ as parameter and with $\Omega=10$.

and $E_{\theta_2}\cos\psi\cos\bar{\theta}_1$ is the component of E_{θ_2} along the short antenna. It is only subject to (13) that the open-circuit voltage V_0^e is proportional to the component of E_{θ_2} along the receiving antenna. It is well to note that (14) applies to a center-loaded antenna with *vanishing currents* at the ends of the antenna. (When multiplied by 2, (14) and (15) are presumably good approximations for short antennas which are so end-loaded by approximately lumped reactances that a sensibly uniform current prevails along the antenna. The present analysis does not include end-loaded antennas.)

The magnitude $|h_e|/\lambda$ and the two components h_e''/λ and h_e'/λ of the complex function

$$h_e/\lambda = (h_e'' + jh_e')/\lambda \qquad (16)$$

as defined in (10) have been computed for a range of values of h/λ ($h/\lambda=0.25$, 0.375, 0.5, 0.5095, 0.625) and of $\bar{\theta}_1$, ($\bar{\theta}_1=0$, 11.5, 23.6, 36.9, and 53.1 degrees so that $\sin\bar{\theta}_1=0$, 0.2, 0.4, 0.6, 0.8) using four different values of Ω ($\Omega=10$, 20, 30, ∞).

Curves of $|h_e|/\lambda$ as a function of h/λ with $\bar{\theta}_1$ as parameter are shown in Figs. 2 to 5; curves of $|h_e|/\lambda$ with $\bar{\theta}_1$ in degrees as variable using h/λ as parameter are given in Figs. 6 to 9. The components h_e''/λ and h_e'/λ are reproduced in Figs. 10 to 15. The long and tedious computations involved in evaluating these curves were carried out with precision for the specified values of h/λ and $\bar{\theta}_1$. Values of Z_{00} were computed directly from (4-10) which neglects terms with the factor $1/\Omega^2$. This involves

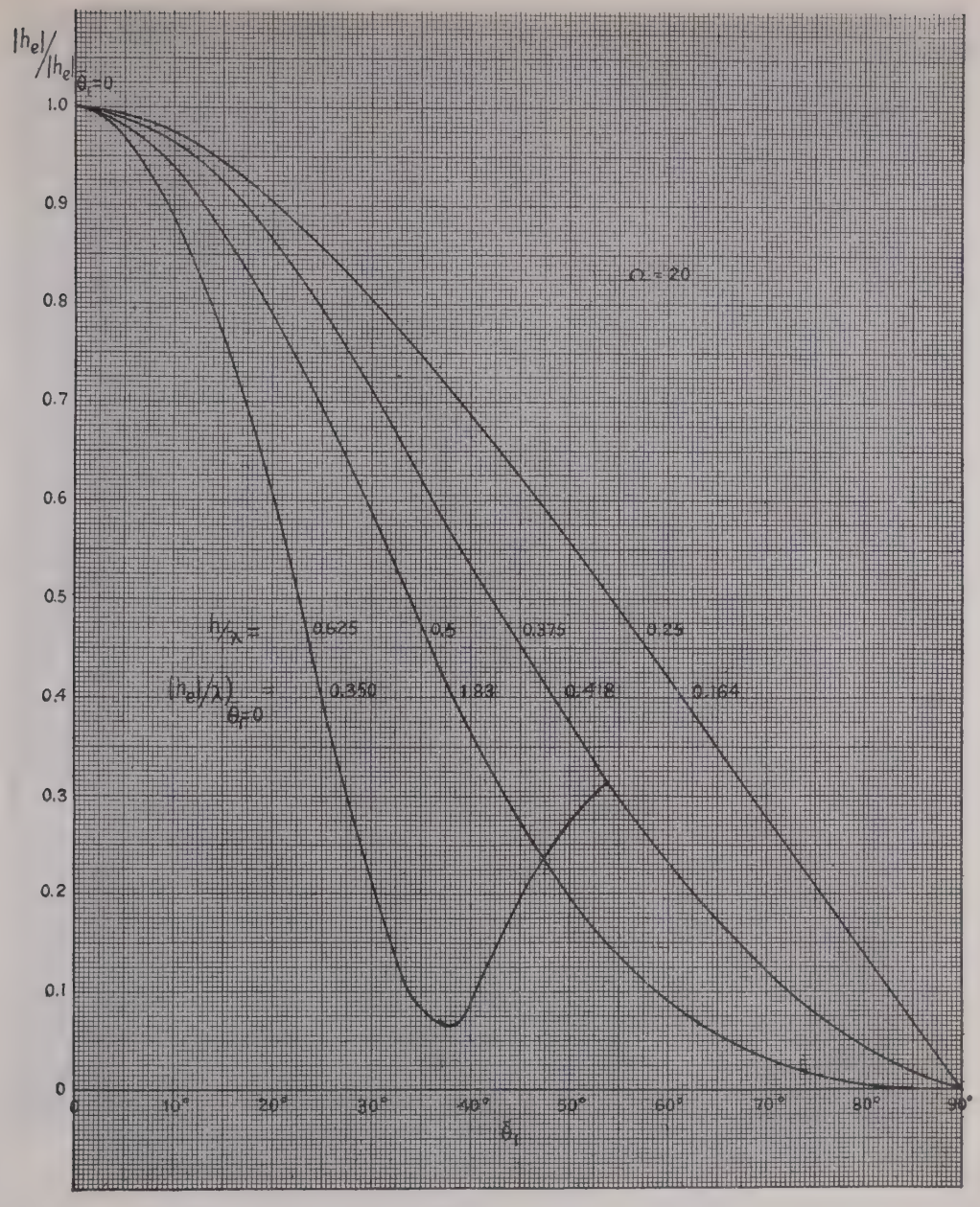


Fig. 7—$|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with h/λ as parameter and with $\Omega = 20$.

no appreciable error for sufficiently large values of Ω except near $\beta h = \pi$ in which case the leading term in the denominator of (4-10) becomes vanishingly small and the term with the factor $1/\Omega$ is left as the largest and only remaining term. In this case to neglect terms with the factor $1/\Omega^2$ and higher powers may involve quite large errors in Z_{00} for the thick antenna with $\Omega = 10$. Because of the fact that the computed results for h_e/λ could not be determined with high precision for h/λ near 0.5 without enormous labor (which most practical applications would not justify), no attempt was made to locate or determine the magnitude of extreme values appearing in h_e/λ or its components with great accuracy. Approximate methods were used which involve possible errors not exceeding 5 per cent. Thus all curves in Figs. 2 to 15 are considerably less accurate very near $h/\lambda = 0.5$ than elsewhere. They should, nevertheless, prove entirely adequate in practical problems even over this part of the range. It is well to note that the curves for an infinitely thin antenna are meaningless for any practical antenna near $h/\lambda = 0.5$, whereas well below this value they are on the whole surprisingly good approximations.

Illustrative Example 1: Antenna in a Linearly Polarized Electric Field.

Consider an antenna for which $\Omega(=2 \ln 2h/a) = 20$ and $h/\lambda = 0.475$. The antenna is used for receiving a 200-megacycle-per-second signal. The electric field strength E in the vicinity of the antenna is 600 microvolts per

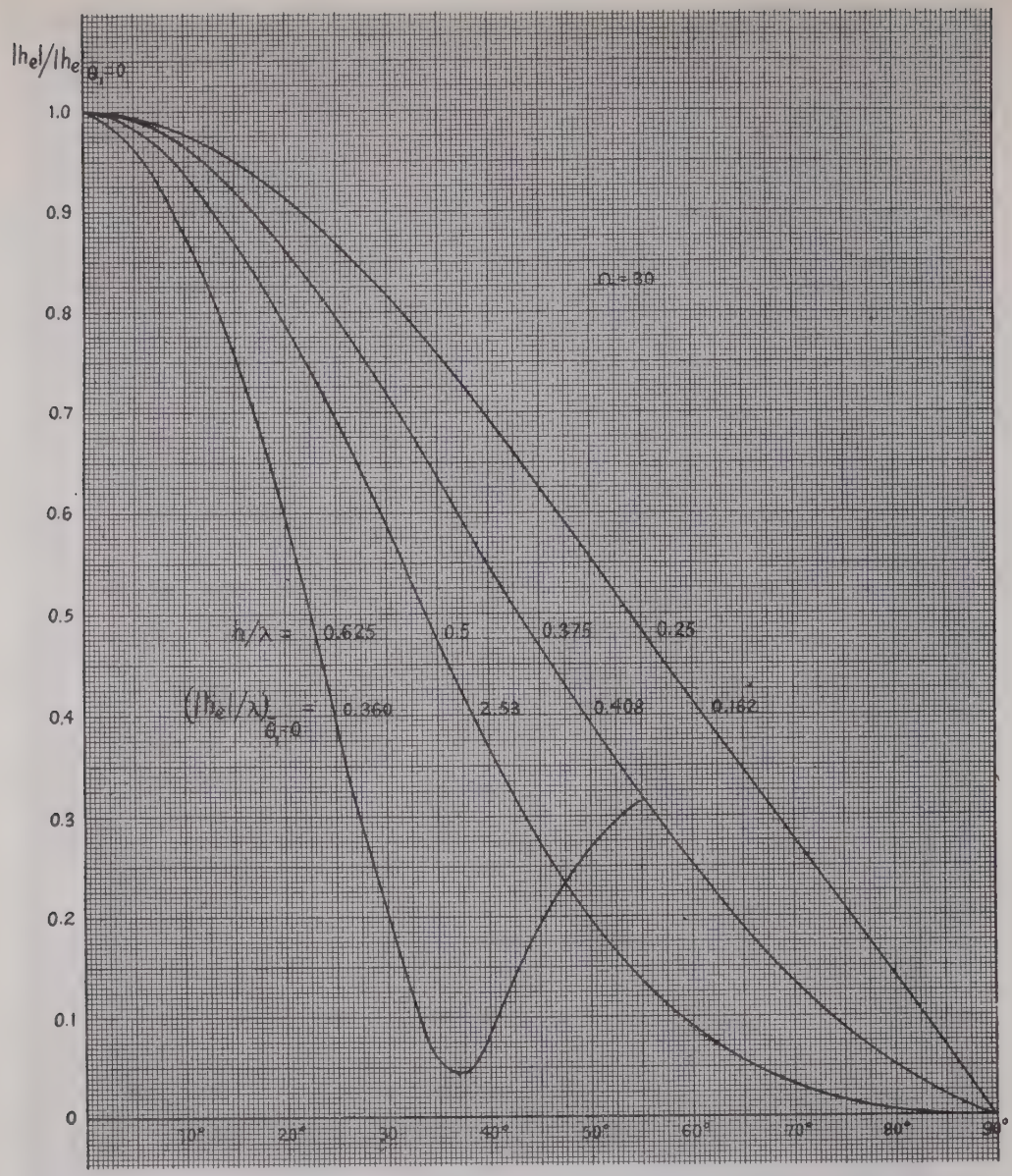


Fig. 8—$|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with h/λ as parameter and with $\Omega=30$.

meter with angles $\psi=33.5$ degrees and $\bar{\theta}_1=36.9$ degrees (refer to Fig. 1). It is desired to find the voltage developed across the resistive component of a load impedance $Z_L=(6-j2.13)10^3$ ohms connected at the center of the antenna.

The input impedance, as calculated from (4-10) is $Z_{00}=(5.85-j2.13)10^3$ ohms for $\Omega=20$ and an antenna half-length, $h=0.7125$ meter.

From Fig. 11 one obtains $h_e'/\lambda=-0.80$; from Fig. 14, $h_e''/\lambda=0.47$. Hence $h_e/\lambda=0.47-j0.80$. (An alternative and simpler method which is adequate when only a single field is involved is to obtain $|h_e|/\lambda=0.93$ from Fig. 3.) Accordingly $h_e=0.705-j1.20$ meters. $V_0^e=2h_eE\cos\psi=(0.705-j1.20)10^{-3}$ volt. $|V_0^e|=2|h_e|E\cos\psi=1.39\times10^{-3}$ volt. The "equivalent" circuit of the receiving antenna from the point of view of the load Z_L consists of $|V_0^e|$ impressed in series with Z_{00} and Z_L. Solving one obtains $|V_L|=0.703$ millivolt.

It is of interest to observe that the same answer would be obtained with an electric field that is circularly polarized in the *plane at right angles to the line joining the distant transmitter to the center of the receiving antenna*, or that is elliptically polarized in the same plane and that is of such magnitude that the component along the antenna is the same as the field strength given for the linearly polarized field.

Illustrative Example 2: Antenna in an Equiphase Surface of an Elliptically Polarized Electric Field.

The receiving antenna of Example 1 is placed perpendicular to the line joining its center with a distant transmitting array which maintains an elliptically polarized field (e.g., a pair of crossed antennas both

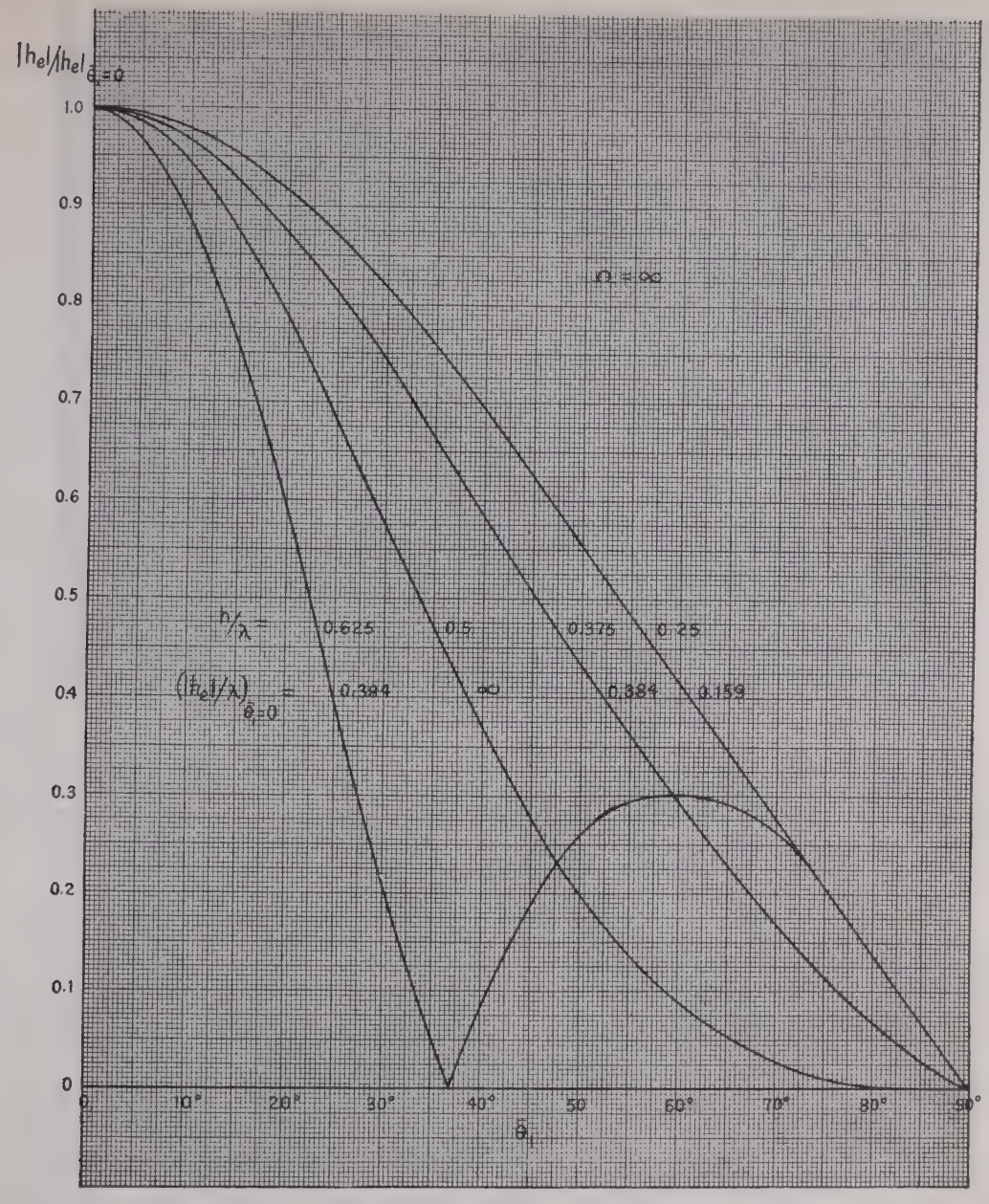


Fig. 9—$|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with h/λ as parameter and with $\Omega = \infty$.

perpendicular to the same line; the antennas carry currents of different amplitudes that are 90 degrees out of phase). In this case the electric vector rotates in such a way that the angle $\bar{\theta}_1$ (Fig. 1) remains zero, while ψ varies continuously from zero to 2π once each period. The electric field may be decomposed into two mutually perpendicular components which remain fixed in space. One of these may be chosen parallel to the receiving antenna ($\psi = 0$) and the other perpendicular ($\psi = 90$ degrees). Both vary periodically, but since the latter contributes nothing to the open-circuit voltage, its magnitude and phase are of no significance whatsoever, and it may be ignored. If the parallel component has a magnitude of 500 microvolts per meter the same numerical answer is obtained as in Example 1.

Illustrative Example 3: Antenna in an Arbitrarily Oriented Elliptically Polarized Field

An elliptically polarized electric field can always be decomposed into two components which are fixed in space and which differ in phase, magnitude, and direction. Actually the most important case is probably that of ionospheric reflection in which the two fixed components of the field rather than their rotating resultant are given. Accordingly this situation will be examined in detail. Suppose the electric field may be described completely in terms of the components E_A and E_B, where E_A has a value of 500 microvolts per meter and is parallel to the receiving antenna so that $\psi_A = 0$ degrees and $\bar{\theta}_{1A} = 0$ degrees, whereas E_B has a value of 600

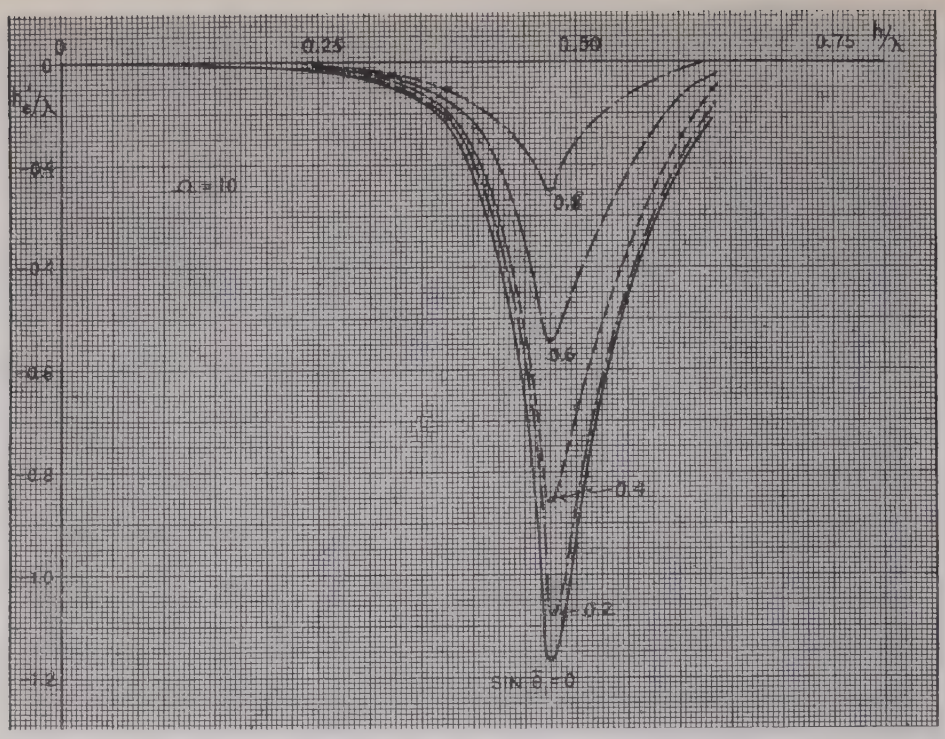


Fig. 10—h_e''/λ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and $\Omega = 10$.

microvolts per meter with angles $\psi_B = 60$ degrees and $\bar{\theta}_{1B} = 53.2$ degrees. Let the effective difference in path lengths back to the transmitter in the particular case of ionospheric reflection (or the phase relations in general) be such that E_A leads E_B by 50 degrees at the receiving antenna.

The complex expressions for the components of the field which lie in the plane containing the receiving antenna, i.e., $E_A \cos \psi_A$ and $E_B \cos \psi_B$, may be written as follows with phases referred to E_A.

$$E_A \cos \psi_A = 500$$
$$E_B \cos \psi_B = 600 \cos 60 \text{ degrees } e^{-j50 \text{ degrees}} = 192.8 - j229.8.$$

Using the same receiving antenna and the same frequency as in Example 1, one obtains from Figs. 11 and 14

$$h_{eA}/\lambda = 0.92 - j1.71; \quad h_{eA} = 1.38 - j2.57;$$
$$h_{eB}/\lambda = 0.23 - j36; \quad h_{eB} = 0.345 - j0.54.$$

Upon substituting in (8) one has

$$V_{0A}{}^e = 2(1.38 - j2.57)(500 + j0) = 1380 - j2570 \text{ microvolts.}$$
$$V_{0B}{}^e = 2(0.345 - j0.54)(192.8 - j229.8) = -115.0 - j366.8 \text{ microvolts.}$$
$$V_0{}^e = V_{0A}{}^e + V_{0B}{}^e = 1265 - j2937 \text{ microvolts}$$
$$|V_0|^e = 3.2 \text{ millivolts.}$$

Using this voltage in the equivalent circuit of the receiving antenna, as in Example 1, one obtains

$$|V_L| = 1.62 \text{ millivolts.}$$

From this illustrative problem it is clear that it is imperative to know the complex expression and not just the magnitude of the effective height of a particular receiving antenna whenever this does not lie in an equiphase plane of an elliptically polarized electric field.

The Reciprocal Theorem and the Distant Field of Driven Antennas

The distant electric field of a symmetrical center-driven linear radiator of length $2h$ (antenna No. 2) may be written in the following way (reference 3, equation (1)):

$$E_{\theta_2} = j60(e^{-j\beta R_0}/R_0) I_{20} V_0(\theta_2). \tag{17}$$

Here $V_0(\theta_2)$ is the "vertical" field characteristic referred to the driving-point current I_{20}. It is defined by

$$V_0(\theta_2) = \frac{\beta}{2} \int_{-h_2}^{+h_2} \frac{I_{2z'}}{I_{20}} e^{-j\beta z' \cos \theta_2} \sin \theta_2 dz'. \tag{18}$$

For an indefinitely thin antenna the distribution of current is given by

$$\frac{I_{2z'}}{I_{20}} = \frac{\sin \beta(h_2 - |z'|)}{\sin \beta h_2}; \quad (a = 0) \tag{19}$$

and (18) integrates into the familiar formula

$$[V_0(\theta_2)]_{a \to 0} = \frac{\cos(\beta h_2 \cos \theta_2) - \cos \beta h_2}{\sin \beta h_2 \sin \theta_2}. \tag{20}$$

If this is compared with the effective half-length (in radians) of the infinitely thin receiving antenna (antenna No. 1) as obtained from (12) using $\theta_1 = (\pi/2) - \bar{\theta}_1$, viz.,

$$[H_e]_{a \to 0} \equiv \left[\frac{2\pi h_1}{\lambda}\right]_{a \to 0} = \frac{\cos(\beta h_1 \cos \theta_1) - \cos \beta h_1}{\sin \beta h_1 \sin \theta_1}. \tag{21}$$

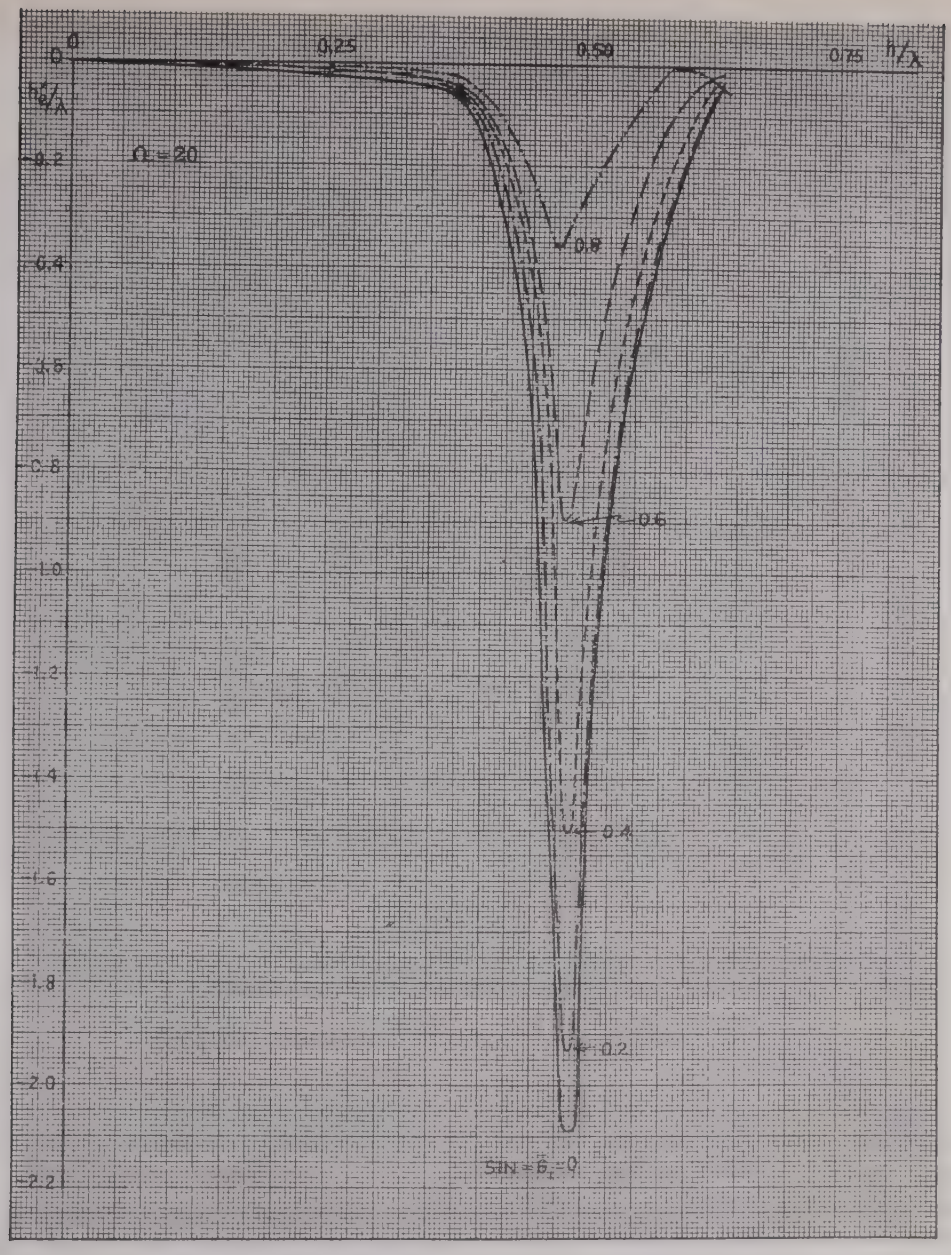


Fig. 11—h_e''/λ as a function of h/λ with $\sin \bar{\theta}_1$ as parameter and $\Omega = 20$.

The two expressions are seen to be identical in form. This could have been predicted directly from the reciprocal theorem, and in itself it contains nothing new. However, if one does not confine one's attention to the idealized case of an infinitely thin antenna, it again follows directly from the reciprocal theorem that the complex "vertical" field characteristic $V_0(\theta_2)$ for a linear radiator of radius a as defined by (18), must be given by (10), multiplied by 2π, and with appropriate subscripts on h and θ after writing $\theta_1 = (\pi/2) - \bar{\theta}_1$. Thus

$$V_0(\theta_2) = \frac{2\pi j Z_{00}}{\Omega R_c \sin \theta_2} \left\{ \frac{[\cos(\beta h_2 \cos \theta_2) - \cos \beta h_2] + 1/\Omega [m_1^I(0) + j m_1^{II}(0)]}{\cos \beta h_2 + 1/\Omega [A_1^I + j A_1^{II}]} \right\}. \quad (22)$$

This is a complex expression. Its magnitude is the complete and accurate formula for the distant field pattern of a center-driven antenna of half-length h and radius a for which terms in $1/\Omega^2$ are negligible. It is equivalent to (18) with the intricate distribution function for I_z, as given by (2-39), substituted under the sign of integration. Accordingly, Figs. 6 to 9 are representations of the magnitudes of the "vertical" field patterns of driven antennas equally well as magnitudes of effective lengths of receiving antennas. They are plotted in the more familiar polar diagram in Figs. 16 and 17 with the value at $\bar{\theta} = \pi/2$ set equal to unity in all cases. These compare extremely well with the corresponding patterns computed from (3-18) using the approximate

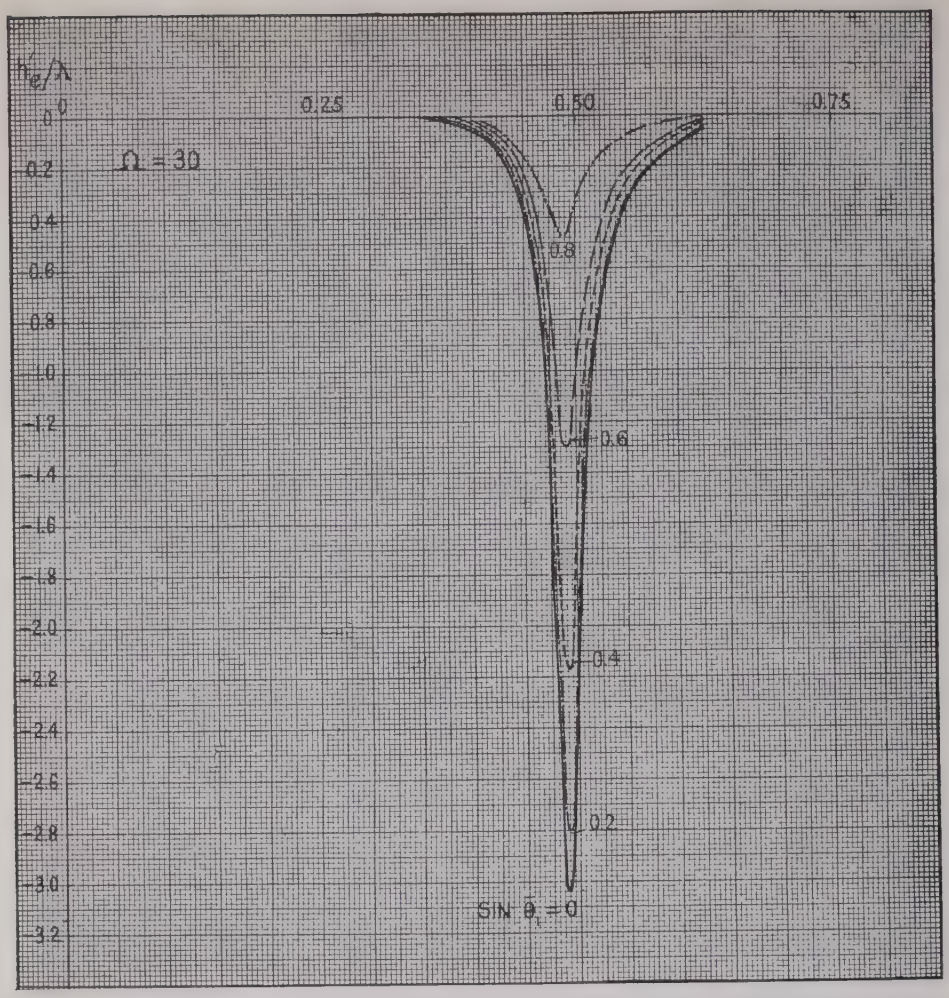


Fig. 12—h_e''/λ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and $\Omega=30$.

distribution of current (3-10). Thus, the present analysis of the receiving antenna constitutes a check on the simpler approximate representation of the distant field of linear radiators carried out in reference 3. The earlier conclusion that the shape of the field pattern for actual antennas does not differ significantly from that of infinitely thin antennas of the same length is again verified. The sharp zeros obtained between ears in field patterns of infinitely thin antennas for which h exceeds $\lambda/2$ are again seen to appear as minima for antennas of nonvanishing radius. In addition an interesting and not insignificant flattening of the pattern is observed for an antenna for which h is fixed at $3\lambda/8$ as the antenna is make thicker. It is seen from Fig. 2-19 that the distribution of current along a moderately thick antenna of this length departs more from the simple sinusoidal distribution than does that for antennas of half-lengths $\lambda/4$ or $\lambda/2$. The amplitude of current in the thick antenna is relatively greater at distances from, and smaller at the center than in an infinitely thin antenna, so that a field resembling that of a somewhat longer antenna with a sinusoidal distribution is obtained as would be expected.

An Approximate Method for Determining the Complex Effective Length of a Receiving Antenna

Since the complex vertical characteristic $V_0(\bar{\theta})$ of a center-driven antenna is the same as the complex effective half-length in radians H_e when the antenna is center-loaded and used for receiving instead of transmitting, it is obviously possible not only to obtain $V_0(\bar{\theta})$ from the computed value of H_e but conversely, to obtain H_e from $V_0(\theta)$. In the preceding section it was pointed out that accurate values of $V_0(\theta)$ for an antenna of nonvanishing radius could be obtained from the curves for $h_e/\lambda(=H_e/2\pi)$, and these were shown to agree well with the approximate values previously computed.[3] It follows directly that approximate values of H_e may be determined from $V_0(\theta)$ obtained by the relatively simple method outlined in reference 3. This made use of the distribution of current given in (2-70), viz.,

$$I_z = I_z'' + jI_0' = I_0''\left(\frac{\cos \beta z - \cos \beta h}{1 - \cos \beta h}\right) + jI_0'\left(\frac{\sin \beta(h - |z|)}{\sin \beta h}\right) \qquad (23)$$

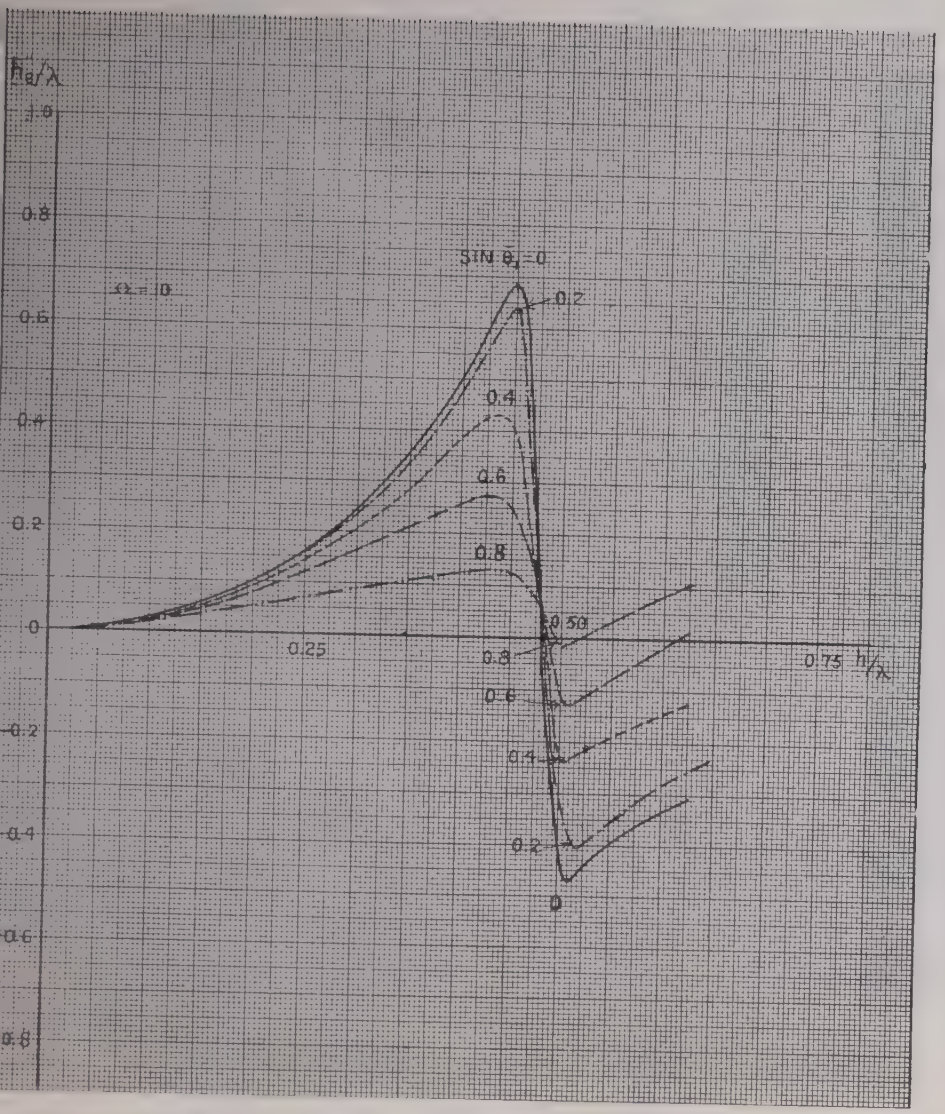


Fig. 13—h_e'/λ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and $\Omega=10$.

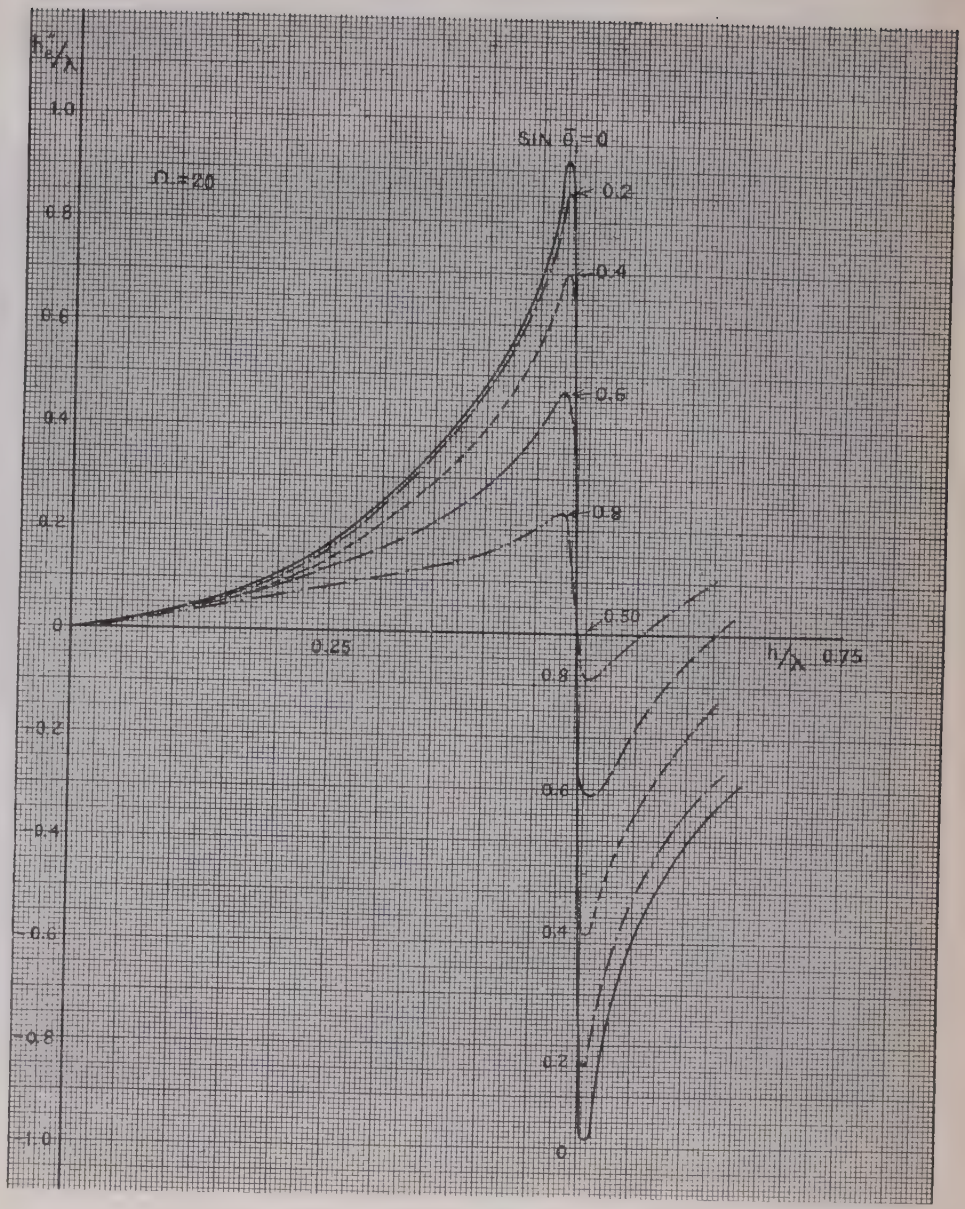


Fig. 14—h_e'/λ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and $\Omega=20$.

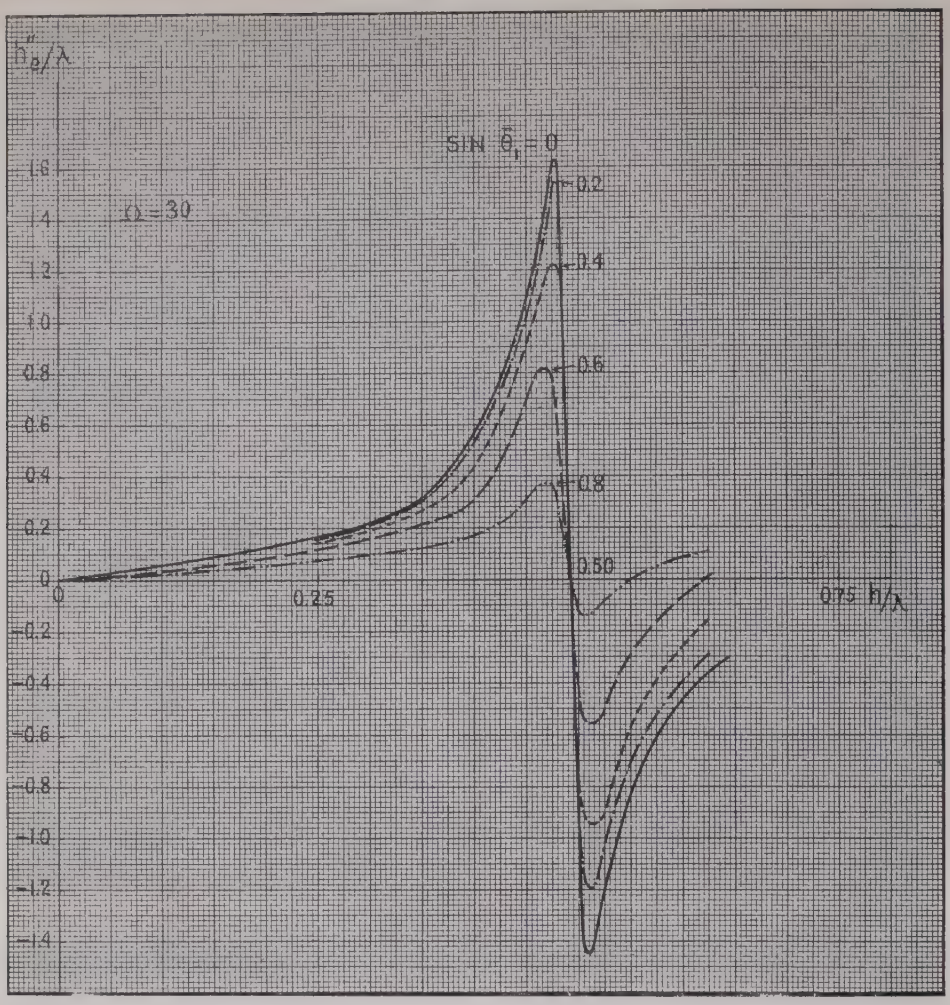


Fig. 15—h_e'/λ as a function of h/λ with sin $\bar{\theta}_1$ as parameter and $\Omega = 30$.

with
$$I_0'' = V_0^e G_{00}, \tag{24}$$
and
$$I_0' = -V_0^e B_{00}, \tag{25}$$
to compute the distant field, (3-11). This is written below in slightly different form:
$$E_\theta^r = 60 I_0 (e^{-j\beta R_0}/R_0) V_0(\theta) \tag{26}$$
with
$$V_0(\theta) = -Z_{00}\left\{ jB_{00}\left[\frac{\cos(\beta h \cos\theta) - \cos\beta h}{\sin\beta h \sin\theta}\right] + \frac{G_{00}}{(1-\cos\beta h)\sin\theta}\right]\sin\beta h \cos(\beta h \cos\theta) - \frac{\cos\beta h \sin(\beta h \cos\theta)}{\cos\theta}\Big]\Big\}. \tag{27}$$

$V_0(\theta)$ is the complex vertical field characteristic of the center-driven antenna. The first term is due to the current in quadrature with the driving potential difference V_0^e; the second term is due to the current in phase with V_0^e. The magnitude $|V_0(\theta)|$ is the actual vertical field pattern proportional to the magnitude of the distant electric field.

Since the complex effective length in radians, viz., $H_e = 2\pi h_e/\lambda$, for an antenna when center-loaded for receiving is the same as $V_0(\theta)$ for the antenna when center-driven, *the above expression is also an approximate formula for* H_e. It may be used as a fair approximation for the magnitude, as a rough estimate for the components of the complex effective length of even moderately thick antennas with appropriate values of Z_{00}, B_{00}, and G_{00} (as determined from data given in reference 4) provided the antennas in question are not near lengths for which sin $\beta h = 0$, or cos $\beta h = +1$. Computation from (27) is very much simpler and considerably less exact than the analysis carried through in determining the curves for h_e/λ described above. For the infinitely thin antenna for which $G_{00} = 0$, $B_{00} = 1/X_{00}$, and $Z_{00} = jX_{00}$, (27) reduces directly to the familiar form (1).

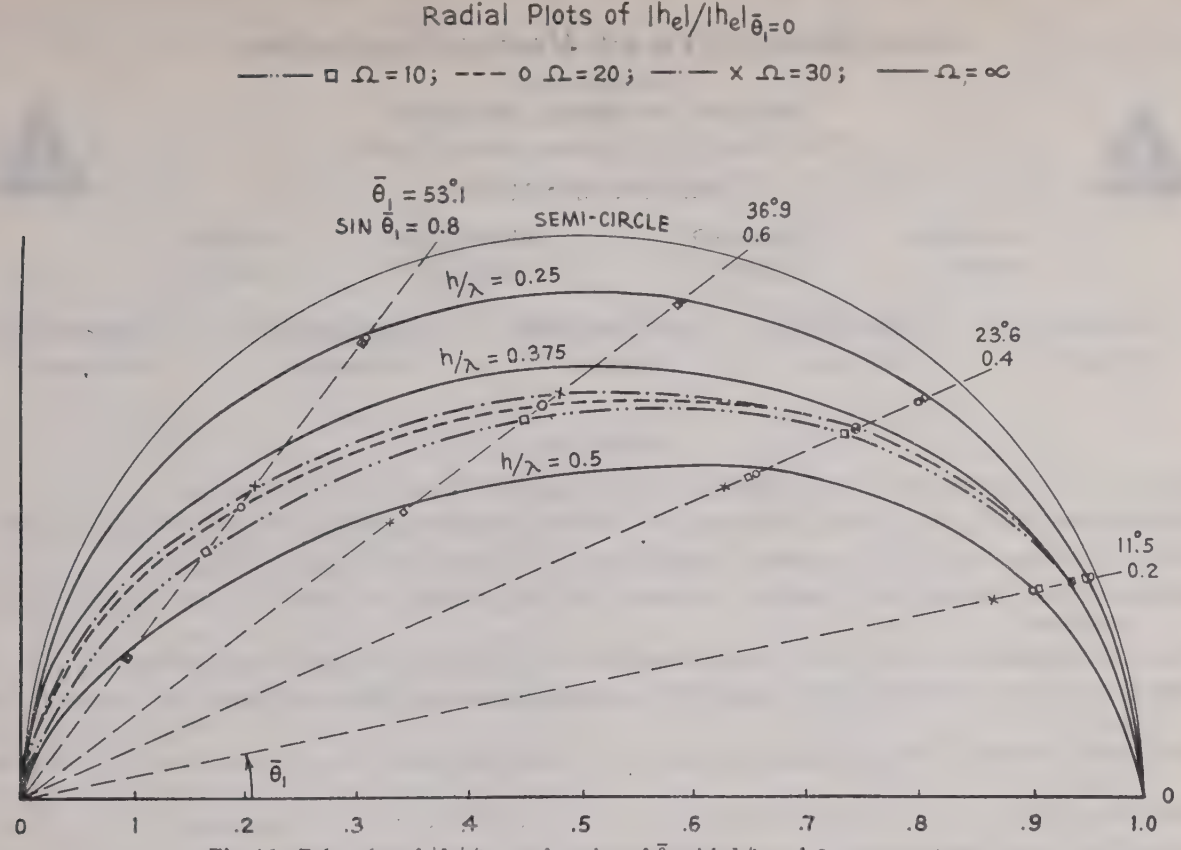


Fig. 16—Polar plot of $|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with h/λ and Ω as parameters.

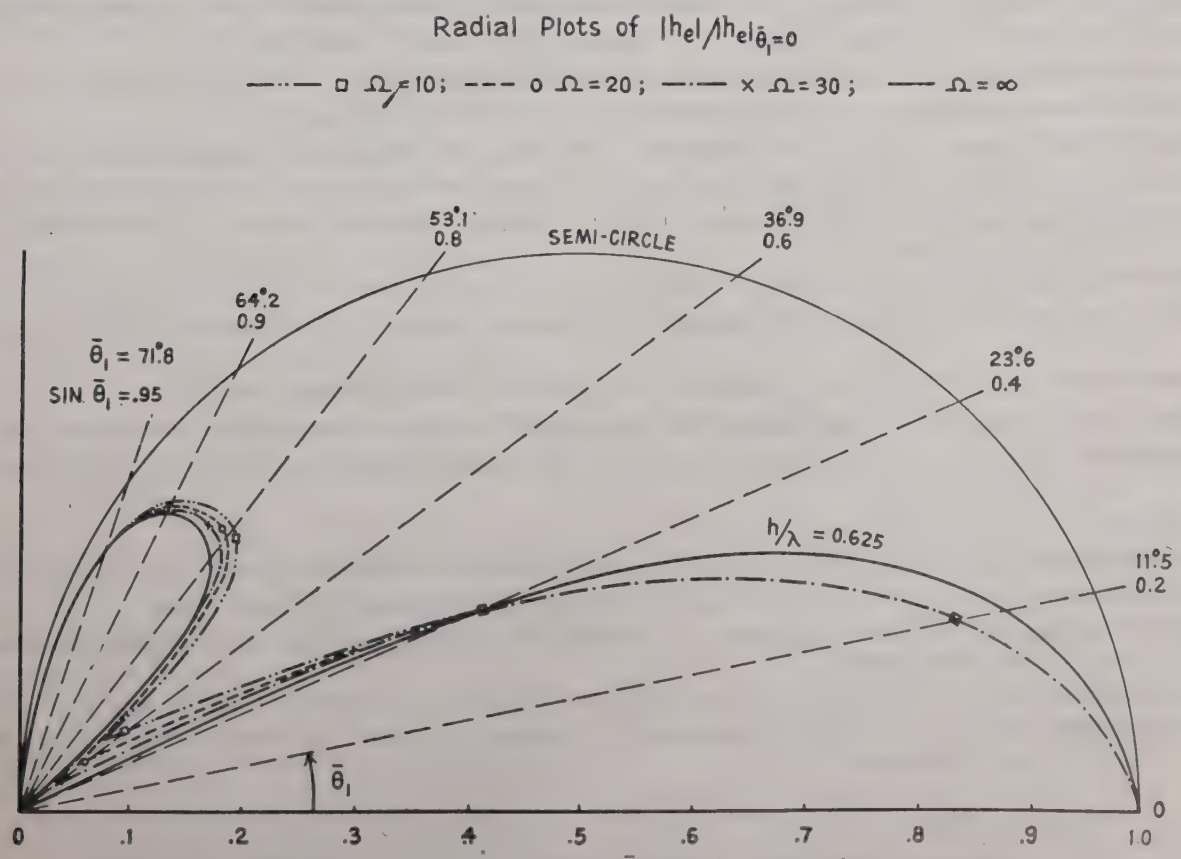


Fig. 17—Polar plot of $|h_e|/\lambda$ as a function of $\bar{\theta}_1$ with $h/\lambda = 0.625$ and Ω as parameter.

THE INSTITUTE OF RADIO ENGINEERS

INCORPORATED

WINTER TECHNICAL MEETING

NEW YORK, N. Y., JANUARY 28 AND 29, 1944

SECTION MEETINGS

ATLANTA	CHICAGO	CLEVELAND	DETROIT
January 21	January 21	January 27	January 18

LOS ANGELES	NEW YORK	PHILADELPHIA	PITTSBURGH	WASHINGTON
January 18	February 2	February 3	February 1	February 14

SECTIONS

ATLANTA—Chairman, C. F. Daugherty; Secretary, Ivan Miles, 554—14 St., N. W., Atlanta, Ga.

BALTIMORE—Chairman, G. J. Gross; Secretary, A. D. Williams, Bendix Radio Corp., E. Joppa Rd., Towson, Md.

BOSTON—Chairman, R. F. Field; Secretary, Corwin Crosby, 16 Chauncy St., Cambridge, Mass.

BUENOS AIRES—Chairman, G. J. Andrews; Secretary, W. Klappenbach, *La Nacion*, Florida 347, Buenos Aires, Argentina.

BUFFALO-NIAGARA—Chairman, Leroy Fiedler; Secretary, H. G. Korts, 432 Potomac Ave., Buffalo, N. Y.

CHICAGO—Chairman, A. B. Bronwell; Secretary, W. O. Swinyard, Hazeltine Electronics Corp., 325 W. Huron St., Chicago, Ill.

CINCINNATI—Chairman, Howard Lepple; Secretary, J. L. Hollis, 6511 Betts Ave., North College Hill, Cincinnati, Ohio.

CLEVELAND—Chairman, A. S. Nace; Secretary, Lester L. Stoffel, 1095 Kenneth Dr., Lakewood, Ohio

CONNECTICUT VALLEY—Chairman, W. M. Smith; Secretary, R. F. Shea, General Electric Co., Bridgeport, Conn.

DALLAS-FORT WORTH—Chairman, H. E. Applegate; Secretary, P. C. Barnes, WFAA-WBAP, Grapevine, Texas.

DETROIT—Chairman, F. M. Hartz; Secretary, E. J. Hughes, 14209 Prevost, Detroit, Mich.

EMPORIUM—Chairman, H. D. Johnson; Secretary, A. Dolnick, Sylvania Electric Products, Inc., Emporium, Pa.

INDIANAPOLIS—Chairman, A. N. Curtiss; Secretary, E. E. Alden, WIRE, Indianapolis, Ind.

KANSAS CITY—Chairman, A. P. Stuhrman; Secretary, R. N. White, 4800 Jefferson St., Kansas City, Mo.

LOS ANGELES—Chairman, Lester Bowman; Secretary, R. C. Moody, 4319 Bellingham Ave., North Hollywood, Calif.

MONTREAL—Chairman, L. T. Bird; Secretary, J. C. R. Punchard, Northern Electric Co., 1261 Shearer St., Montreal, Que., Canada.

NEW YORK—Chairman, Lloyd Espenschied; Secretary, J. E. Shepherd, 111 Courtenay Rd., Hempstead, L. I., N. Y.

PHILADELPHIA—Chairman, W. P. West; Secretary, H. J. Schrader, Bldg. 8, Fl. 10, RCA Manufacturing Co., Camden, N. J.

PITTSBURGH—Chairman, B. R. Teare; Secretary, R. K. Crooks, Box, 2038, Pittsburgh, 30, Pa.

PORTLAND—Chairman, B. R. Paul; Secretary, W. A. Cutting, c/o U. S. Civil Aeronautics, Box 1807, Portland, Ore.

ROCHESTER—Chairman, O. L. Angevine, Jr.; Secretary, G. R. Town, Stromberg-Carlson Telephone Manufacturing Co., Rochester, N. Y.

ST. LOUIS—Chairman, N. J. Zehr.

SAN FRANCISCO—Chairman, Karl Spangenberg; Secretary, David Packard, Hewlett-Packard Co., Palto Alto, Calif.

SEATTLE—Chairman, L. B. Cochran; Secretary, H. E. Renfro, 4311 Thackeray Pl., Seattle, Wash.

TORONTO—Chairman, T. S. Farley; Secretary, J. T. Pfeiffer, Erie Resistor of Canada, Ltd., Terminal Warehouse Bldg., Toronto, Ont., Canada.

TWIN CITIES—Chairman, E. S. Heiser; Secretary, B. R. Hilker, KSTP, St. Paul Hotel, St. Paul, Minn.

WASHINGTON—Chairman, J. D. Wallace; Secretary, F. W. Albertson, c/o Dow and Lohnes, E Street between 13th and 14th Sts., Washington, D. C.

1944 I.R.E. Winter Technical Meeting
January 28 and 29, 1944
Hotel Commodore, New York, N. Y.

The Board of Directors of the Institute has approved the 1944 Winter Technical Meeting to take place on Friday and Saturday, January 28 and 29, 1944 at the Hotel Commodore in New York City.

This two-day meeting will be devoted to the domestic and international aspects of present and postwar radio-and-electronic engineering and thus will be of timely interest to the entire Institute membership. In addition to a number of outstanding technical papers, the program will contain several features including participating activities by the Federal Communications Commission, a display of captured enemy radio equipment, and discussion on the newly established Radio Technical Planning Board.

The Institute has been requested to co-operate in conserving the nation's transportation facilities and consequently is confining the mailing of the Winter Technical Meeting notices to members residing east of the Mississippi River, and those in the St. Louis and Twin Cities Sections.

As this issue of the PROCEEDINGS goes to press, the program has not been completed but the following outline of the sessions can be given at this time:

Friday, January 28—Hotel Commodore

9:00 A.M.—Registration

10:30 A.M.—Address of Welcome—B. E. Shackelford, Chairman, 1944 Winter Technical Meeting

10:40 A.M.—L. P. Wheeler Presiding

10:45 A.M.—Ceremony of "Passing the Gavel" from Retiring President Wheeler to Incoming President Turner

11:00 A.M.—Annual Meeting of the Institute, H. M. Turner Presiding. Amendment of Institute's Charter

11:15 A.M.—Dr. Wheeler Resumes Chairmanship for the Session of Technical Papers

12:20 P.M.—Session Adjournment

12:30 P.M.—President's Luncheon—Professor Turner

2:30 P.M.—Symposium—Haraden Pratt Presiding
"Work of the Radio Technical Planning Board," by W. R. G. Baker, Chairman of RTPB; Several Panel Chairmen

4:30 P.M.—Mr. Pratt Resumes Chairmanship for the Session of Technical Papers

5:30 P.M.—Session Adjournment

7:00 P.M.—I.R.E. Banquet (Informal)
George Lewis, Master of Ceremonies
Awards, Presented by Professor Turner
1944 Medal of Honor—to Haraden Pratt
1943 Morris Liebmann Memorial Prize—to W. L. Barrow
1944 Fellowship Awards—to S. L. Bailey, C. R. Burrows, M. G. Crosby, C. B. Feldman, Harry Diamond, Keith Henney, D. O. North, K. A. Norton, S. W. Seeley, D. B. Sinclair, Leo Young
Annual Address of Retiring President
L. P. Wheeler
Prominent Speaker on Timely Subject (to be announced)

Saturday, January 29—Hotel Commodore

10:00 A.M.—Symposium—H. M. Turner Presiding
"Engineering Work of the Federal Communications Commission," by E. K. Jett, Chief Engineer, Federal Communications Commission

"Timely Broadcast Matters" by G. P. Adair, Assistant Chief Engineer of the Federal Communications Commission

"Police Aviation and Maritime Services," by W. N. Krebs, Chief of the Safety and Special Services Division, Federal Communications Commission

"International Point-to-Point and Allocation Problems," by P. F. Siling, Chief of International Division, FCC Engineering Department

---

12:30 P.M.—Students' Luncheon

2:30 P.M.—Technical Session—Lloyd Espenschied Presiding

U OF I

It is also expected to have prominent American and foreign authorities outline radio engineering in their respective countries.

4:30 P.M.—Final Adjournment

I.R.E. Sections Committee Meeting
January 27, 1944

The Annual Meeting of the Committee on Sections will be held at 2:30 P.M. on Thursday, January 27, at the Hotel Commodore, New York City.

At this meeting a number of questions of interest to members and to Sections will be discussed. The agenda contains, besides a summary of Section reports, the question of Institute relation to the education of its future members, viz., college courses for radio and communication, and also the subject of lecture courses for present members. The large subject of broadening the scope of Institute activities will be discussed. This includes a publication program involving papers of value to design and operating engineers, review papers, etc., whose purpose is to keep engineers up-to-date; also more news and notes, both personal and technical that do not justify "papers." The problems of a bureau to handle papers or lectures, and possibly the lecture courses, will be mentioned. What these will require from the membership in the line of increased expense will be brought up. The need for permanent Institute headquarters will be outlined, and the question of handling its financing will be discussed. The opportunity will be presented to discuss other subjects of lesser importance.

All members of the Institute are invited to attend, to hear, and to contribute to the discussion.

It is suggested that engineers, who plan to attend the joint evening meeting with the American Institute of Electrical Engineers, advance their arrival time to attend this meeting of the Sections Committee.

Joint Evening Session With A.I.E.E.
January 27, 1944

Of interest to the I.R.E. membership are the arrangements which have been made with the American Institute of Electrical Engineers for a joint evening meeting on Thursday, January 27, 1944, to be held in the Auditorium of the Engineering Societies Building, 33 West 39 Street, New York City. Major General R. B. Colton, of the Signal Corps, will talk about enemy communication equipment and will display some of the captured apparatus. (It is planned to move this exhibit to the Hotel Commodore on Friday afternoon.)

The A.I.E.E. is holding its technical meeting during the week of January 24, 1944 and, for the convenience of I.R.E. members who desire to attend, their communications papers will be concentrated during the afternoon session of January 27, 1944.

The program for the latter date follows.

Thursday, January 27, 1944
33 West 39 Street, New York, N. Y.

2:00 P.M.—A.I.E.E. Session

Five Electronics and Communication Papers Given Below:

"A Short-Cut Method of Estimating the Telephone Interference Factor of Power Systems with Rectified Load," by C. W. Frick

"Crossbar Toll Switching System," by L. G. Abraham, A. J. Busch, and F. F. Shipley

"Automatic Ticketing of Telephone Calls," by O. A. Friend

"Electronically Controlled Dry-Disk Rectifier," by Allen Rosenstein and H. N. Barnett

"Rectifier Circuit Duty," by C. C. Herskind

8:00 P.M.—Joint A.I.E.E.-I.R.E. Meeting

"Enemy Army Communications Equipment," by Major-General R. B. Colton, Signal Corps, United States Army. Exhibition of Captured Apparatus

Hotel Reservations

Arrangements have been made with the Hotel Commodore, the location of the 1944 Winter Technical Meeting, for accommodating the out-of-town members of the Institute. However, because of crowded conditions, the hotel has requested that room reservations be made in advance and direct with the hotel (and not through the Institute office) as promptly as possible. The hotel cannot guarantee reservations within a week of the meeting dates.

As part of the special arrangement with the hotel, requests for room reservations should mention the members' affiliation with the Institute and the I.R.E. Winter Technical Meeting. Such mention will enable the hotel to give prompt and preferred attention to the reservations from Institute members.

Make your reservations early because the number of available rooms is limited. Address your request to the Hotel Commodore, New York City.

General Committee for
1944 Winter Technical Meeting

B. E. Shackelford, *Chairman*
Austin Bailey, *Vice Chairman*

I. S. Coggeshall	F. A. Gunther
E. J. Content	George Lewis
W. B. Cowilich	F. B. Llewellyn
H. F. Dart	J. R. Poppele
G. A. Downsbrough	Haraden Pratt
Alfred N. Goldsmith	H. A. Wheeler

L. J. Woods

Board of Directors

The regular meeting of the Board of Directors took place on November 3, 1943, and was attended by L. P. Wheeler, president; F. S. Barton, vice president; S. L. Bailey, W. L. Barrow, I. S. Coggeshall, W. L. Everitt, H. T. Friis, Alfred N. Goldsmith, editor; G. E. Gustafson, O. B. Hanson, R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; F. E. Terman, B. J. Thompson, H. M. Turner, H. A. Wheeler, W. C. White, and W. B. Cowilich, assistant secretary.

The actions of the Executive Committee, taken at its October 5, 1943 meeting, were ratified.

These applications for membership were unanimously approved:

For transfer to Senior Member grade, A. B. Bronwell, W. M. Breazeale, R. D. Chipp, D. W. Epstein, W. T. Gibson, L. G. Hector, A. B. Hunt, D. W. Jenks, R. W. P. King, F. A. Lidbury, F. H. Nicoll, J. W. Paddon, Dale Pollack, R. B. Shanck, and McMurdo Silver; for admission to Senior Member grade, P. L. Barker, S. J. Levy, H. J. McCreary, J. R. Moore, and R. M. Ryder; for transfer to Associate grade, 17; Associate grade, 133; and, Student grade, 70.

A number of Bylaws amendments, necessitated by Constitutional changes, were adopted. Other amendments of the Bylaws, relative to the organization and duties of the Investment Committee, were discussed.

The report of the Tellers Committee was received and the following were declared elected:

President, 1944: H. M. Turner.

Vice President, 1944: R. A. Hackbusch.

Directors, 1944–1946: R. F. Guy, L. C. F. Horle, W. C. White.

Mr. Coggeshall, chairman of the Public Relations Committee, and Assistant Secretary Cowilich were requested to issue an immediate press release on the election of the foregoing officers.

These members were elected to serve on the Appointments Committee: B. J. Thompson, chairman; W. L. Barrow, F. B. Llewellyn, H. M. Turner, and H. A. Wheeler.

Secretary Pratt, as Institute Representative on the Radio Technical Planning Board, gave a report on the activities of that body and indicated that the American Institute of Electrical Engineers and the National Electrical Manufacturers Association were recently elected as contributing sponsors.

The election of Assistant Secretary Cowilich as Secretary of the Radio Technical Planning Board was sanctioned.

On recommendation of the Executive Committee, a resolution containing the amendments to the Institute's Certificate of Incorporation, was unanimously adopted.

Action took place on matters relative to the Institute's investments, reported by Treasurer Heising, including the appointment of General Counsel Zeamans to the Investment Committee.

At his request, the Board of Directors granted an audience to Professor E. H. Armstrong for a presentation of his viewpoint on certain technical and procedural matters.

As Chairman of the Office-Quarters Committee, Treasurer Heising gave a progress report on the investigation of leasing or buying larger office quarters. The subject was discussed, and a general plan for raising funds for a building endowment was favorably received.

H. A. Wheeler, responsible for the Institute's convention activities, described the progress being made in the plans for the 1944 Winter Technical Meeting, which will be held on January 28 and 29, 1944, in New York. It was decided to join with the American Institute of Electrical Engineers for an evening meeting on January 27, 1944.

President Wheeler announced the appointment of the following personnel of the Institute's Committee on Professional Representation: H. A. Wheeler, chairman; E. F. Carter, C. C. Chambers, J. V. L. Hogan, and W. C. White. Mr. Wheeler reported on certain recommendations of this committee, pertaining to activities which would promote the professional standing of radio engineers.

H. F. Dart was unanimously elected as an additional member of the Institute's Committee on Professional Representation.

The New York Section was complimented on its enterprise in carrying out the publication of the comprehensive New York Technical Meetings Calendar for the last few months.

The appointment of J. F. Johnson to the Membership Committee was approved.

President-elect Turner was added to the Executive Committee, effective with the next meeting of that group.

The subject of an office-pension policy, introduced by President Wheeler, was discussed and the Executive Committee was authorized to make a further study of the matter.

Executive Committee

The Executive Committee met on November 2, 1943, and those present included L. P. Wheeler, president; Alfred N. Goldsmith, editor; R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; H. A. Wheeler, and W. B. Cowilich, assistant secretary.

Approval was granted to the following applications for membership, and to the recommendation that the Board of Directors advance those of Member grade to the new Senior Member status:

Transfer to Senior Member grade, A. B. Bronwell, W. M. Breazeale, R. D. Chipp, D. W. Epstein, W. T. Gibson, L. G. Hector, A. B. Hunt, D. W. Jenks, R. W. P. King, F. A. Lidbury, F. H. Nicoll, J. W. Paddon, Dale Pollack, R. B. Shanck, and McMurdo Silver; admission to Member grade, P. L. Barker, S. J. Levy, H. J. McCreary, J. R. Moore, and R. M. Ryder; transfer to Associate grade, 17; Associate grade, 133; and, Student grade, 70.

Several matters pertaining to office personnel, including an office-pension policy, were given consideration. Assistant Secretary Cowilich reported on the extent of the overtime work in the office during the month of October.

Treasurer Heising, as chairman of the Office-Quarters Committee, gave a report on the possibilities of obtaining larger office quarters on rental or ownership bases. In the discussion of the situation, consideration was also given to a building-endowment fund.

Treasurer Heising reviewed the proposals made by General Counsel Zeamans relative to amending the Institute's Certificate of Incorporation. The proposals were discussed and followed by a recommendation to the Board of Directors.

The appointment of J. F. Johnson to the Membership Committee was recommended to the Board of Directors.

Attention was called by Editor Goldsmith to the preparation of another appeal to the War Production Board for the purpose of obtaining the 1944 paper allotment for the PROCEEDINGS. Complaints concerning the poor legibility resulting from lighter-weight paper in the PROCEEDINGS were also noted.

Action was taken on other matters pertaining to the PROCEEDINGS, as reported by Editor Goldsmith.

It was stated by Editor Goldsmith that a request for technical information on transmitting equipment, from an Argentine standardizing body, had recently been received and that relevant material has been supplied in reply.

The publication of the report, "Temporary Facsimile Test Standards," was recommended to the Board of Directors.

President Wheeler announced that Assistant Secretary Cowilich had recently been appointed Secretary of the Radio Technical Planning Board.

Treasurer Heising, as chairman of the Investment Committee, reviewed the report on investments to be presented to the Board of Directors.

On the subject of the 1944 Winter Technical Meeting, to be held on January 28 and 29 in New York, H. A. Wheeler gave a short account of the preliminary plans.

President Wheeler called attention to the New York Technical Meetings Calendar, being published by the New York Section, which was discussed and followed by a recommendation to the Board of Directors.

The matter of clearing members of certain technical committees, for consideration of classified material, introduced by Dr. Llewellyn, was given consideration.

At the suggestion of Dr. Llewellyn, the Institute's affiliation with the Engineers' Council for Professional Development was discussed.

Sample shades of the light blue, the emblem color for the new Member grade, were examined.

President Wheeler referred to the recent letter from the National Roster of Scientific and Specialized Personnel, relative to the new War Manpower Commission procedures to bring about the most effective utilization of scientific and professional personnel.

Approval was given to printing the revised Constitution as a separate publication and to making copies of it available to the membership.

Correspondence

The form of the correction term used by Pieracci[1] can be arrived at from mathematical considerations. It will be remembered that in a phase-shift modulator the carrier before multiplication is

$$e_c = E(t) \sin [\omega_0 t + \tan^{-1} p \sin \lambda t]. \quad (1)$$

The maximum linear phase shift is determined by p, which for 5 per cent total distortion[2] is $p=0.454$. This corresponds to 25.5 degrees. Pieracci showed that if p were made a function of time then the maximum phase shift for the same distortion could be practically doubled. By synthesis he arrived at the form for $p(t)$ to give a maximum linear phase shift of 60 degrees. From his results

$$p(t) = \frac{1}{0.765 + 0.188 \cos 2\lambda t}. \quad (2)$$

If the argument of the arc tan in (1) were of the form tan $(p \sin \lambda t)$ then the phase variation with modulation would be entirely linear since

$$\tan^{-1} \tan (p \sin \lambda t) = p \sin \lambda t.$$

We now ask ourselves what is the closest approximation to tan $(p \sin \lambda t)$. For this purpose we make use of the expansion[3]

$$\tan x = x \frac{[1-(x/\pi)^2][1-(x/2\pi)^2] \cdots}{[1-(2x/\pi)^2][1-(2x/3\pi)^2] \cdots} \quad (3)$$

also[4]

$$\left.\begin{aligned} \sum_1^\infty 1/n^2 &= \pi^2/6 \\ \sum_0^\infty 1/(1+2n)^2 &= \pi^2/8 \end{aligned}\right\} \quad (4)$$

whence

$$\tan x \cong x \frac{1 - \frac{1}{6}x^2 + \cdots}{1 - \frac{1}{2}x^2 + \cdots} \quad (5)$$

and approximately

$$\tan x \cong x \frac{1}{1 - \frac{1}{2}x^2},$$

$$\tan (p \sin \lambda t) \cong p \sin \lambda t \frac{1}{1 - \frac{1}{2}p^2 \sin^2 \lambda t}$$

$$\cong p \sin \lambda t \frac{1}{(1 - \frac{1}{4}p^2) + \frac{1}{4}p^2 \cos 2\lambda t}. \quad (6)$$

Substituting (6) for $p \sin \lambda t$ in (1) should result in improved phase linearity with modulation. The instantaneous phase increment becomes

$$\Delta\phi = \tan^{-1} \frac{p \sin \lambda t}{(1 - \frac{1}{4}p^2) + \frac{1}{4}p^2 \cos 2\lambda t} \quad (7)$$

which is the form used by Pieracci. If we choose a maximum phase displacement of 60 degrees when $\sin \lambda t = 1$ then $p=0.948$ whence

$$\Delta\phi = \tan^{-1} \frac{\sin \lambda t}{0.814 + 0.237 \cos 2\lambda t} \quad (8)$$

which is in fair agreement with (2).

D. L. Jaffe
Raytheon Manufacturing Co.
Waltham, Mass

[1] Roger J. Pieracci, "A stabilized frequency-modulation system," Proc. I.R.E., vol. 30, pp. 76–81; February, 1942.
[2] D. L. Jaffe, "Armstrong's frequency modulator," Proc. I.R.E., vol. 26, pp. 475–482; April, 1938.
[3] B. O. Pierce, "A short table of integrals," third revised edition, Ginn and Co., New York, N. Y., forms 816 and 817.
[4] L. B. W. Jolley, "Summation of Series," Chapman Hall Ltd., 1925.

Rochester Fall Meeting—1943

The 1943 Rochester Fall Meeting was held on November 8 and 9 at the Sagamore Hotel, Rochester, New York. It consisted of six sessions and featured a large exhibit of captured enemy radio and telephone equipment made available through the courtesy of the United States Army Signal Corps.

F. S. Barton, vice president of the Institute of Radio Engineers; R. A. Hackbush, vice-president-elect of the I.R.E.; O. L. Angevine, chairman of the Rochester Section; W. R. G. Baker, director of engineering of the Radio Manufacturers Association; and R. M. Wise, Sylvania Electric Products, Inc., presided over the technical sessions.

The annual Rochester Fall Meeting dinner was attended by 270. George Lewis, International Telephone and Radio Manufacturing Company, served as toastmaster. Among the speakers were R. K. Gessford, Sylvania Electric Products, Inc.; H. J. Klumb, Rochester Fall Meeting Committee; J. W. Van Allen, Radio Manufacturers Association; R. A. Hackbush, vice-president-elect of the I.R.E.; and Bond Geddes, Radio Manufacturers Association.

A total of 524 registered for all sessions of this meeting.

Books

Practical Radio Communication, by Arthur R. Nilson and J. L. Hornung.

Published (1943) by McGraw-Hill Book Co., 330 West 42nd St., New York 18, N. Y. 914 pages+13-page index +xvii pages. 540 figures. $6\frac{1}{4} \times 9\frac{1}{4}$ inches. Price, $5.00.

This book is intended to cover two phases of radio communication:

1—Fundamentals, which are included in chapters one to eight.
2—Operation and maintenance, which are covered from chapter nine to the end of the book.

The chapter headings give a fairly accurate idea of the scope of the book. These are:

Chap. 1. Direct-Current Electricity and Magnetism
Chap. 2. Alternating-Current Electricity
Chap. 3. Introduction to Radio Tubes and Circuits
Chap. 4. Transmitting-Circuit Principles
Chap. 5. Receiving-Circuit Principles
Chap. 6. Electronic Power Supplies
Chap. 7. Antennas and Wave Propagation
Chap. 8. Frequency Modulation
Chap. 9. Aviation Radio Communication
Chap. 10. Broadcast Studio
Chap. 11. Broadcast Control Room
Chap. 12. Broadcast AM Transmitters
Chap. 13. Marine Radio Transmitters
Chap. 14. Marine Radio Receivers
Chap. 15. Marine Automatic Radio Alarms
Chap. 16. Marine Radio Direction Finders
Chap. 17. Motors and Generators
Chap. 18. Storage Batteries
Appendix

The first eight chapters are covered in the first 429 pages. This part of the book is poorly done. Many statements are incorrect as also are many of the formulas. These errors are too numerous to permit more than mere mention of a few. Besides the errors the first part of the book would be much improved if more recent literature had been used or quoted to a greater extent.

The authors have chosen to run counter to convention and take the direction of electron flow as the direction of current. This is certainly of no help to the student who may have already learned to use the right-hand rule.

Some of the formulas quoted are not even approximately correct. It is evident that this was not done for the sake of simplicity because the correct formulas are quite as simple and easy to understand.

In several instances erroneous statements are to be found. For example, on page 117 one reads, "It may be seen that if the number of turns in the radio-frequency transformer is larger in the secondary than in the primary, a greater e.m.f. will be applied across the secondary at the points *E*." The transformer referred to is an air-core radio-frequency type. The statement checks with formula number 5 listed on the next page. This formula is also entirely wrong.

Many statements are not clear or are

misleading. One outstanding example appears on page 72 regarding inductance, which reads in part as follows: "... expressed in electromagnetic units measured in centimeters which for practical purposes are called henrys, in honor of the discoverer ..." Not until the next page does the reader find that 10^9 centimeters equals one henry. Another sample of careless statement is found on page 63: "These losses are known as hysteresis and eddy currents." The work "efficiency" is loosely used through the text. It is taken to mean almost anything of the nature of a ratio.

Occasionally new terms or ideas are introduced without explanation. For instance, table 1-A, page 12, gives the resistance of a circular mil-foot of several different kinds of wire. What is meant by a circular mil-foot is not explained. The term magnetomotive force first appears on page 39 in its abbreviated form (m.m.f.). Not until two pages later is the subject of magnetism taken up. On page 185, the operator j, $(\sqrt{-1})$ occurs suddenly without explanation. From this point on it is used occasionally throughout the book. The reviewer was able to find only one explanation of what j meant in the entire book, and that was in the list of abbreviations on page xx where: "$j=\sqrt{-1}$ (operating vector)" is found.

These examples are only a few of the many defects of the first eight chapters. One who uses this book would be wise to regard all of this section of the book as of questionable accuracy until verified from some other source.

Beginning with chapter nine, page 430, the quality of the book improves remarkably. Here it is that the authors get busy with their main theme which is the operation and maintenance of modern commercial equipment. Many well-chosen examples of the various types of broadcast, aviation, and marine communication and navigation gear are presented. Complete circuit diagrams showing control as well as radio and audio circuits are accompanied by good word descriptions of their operation. Photographs are used extensively to show the appearance of the various types of equipment. This section of the book (chapter nine to the conclusion) is well done.

The text might better be titled "The Operation and Maintenance of Radio Communication Equipment." The book is written at trade-school level and is intended for operating personnel. It would be a good book for this purpose if the first eight chapters were not so carelessly handled.

L. M. Clement
The Crosley Corporation
Cincinnati, Ohio

Reference Data for Radio Engineers

Compiled and published (1943) by Federal Telephone and Radio Corporation, 67 Broad Street, New York, N. Y. 200 pages+3-page index. $5\frac{3}{4}\times8\frac{3}{4}$ inches. Clothbound, price $1.00 or $0.75 in lots of twelve or more.

This is a follow-up edition of an extremely handy reference book prepared for governmental agencies and educational institutions covering material of value to engineers and operating technicians, and concerns a great many communication problems.

The material has apparently been selected with great care, by those well acquainted with the requirements of practicing engineers, and contains much information not found in any other book. In addition, many of the commonly referred-to tables and charts are found therein.

Among the many subjects taken up, the pages devoted to ultra-high-frequency and its propagation are of great utility. The book can be obtained at so modest a cost that it doubtless will receive a warm welcome. A complete list of sections follows, showing the scope of the undertaking: General Engineering Tables, Engineering and Material Data, Audio and Radio Design—General, Rectifiers, Vacuum Tubes and Amplifiers, Telephone Transmission, Radio-Frequency Transmission Lines, Radio Propagation and Antennas, Noise and Noise Measurement, Non-Sinusoidal Waveforms, Dimensional Expressions, Mathematical Formulas and General Information, and Mathematical Tables.

R. R. Batcher
St. Alban's, L. I., N. Y.

Physik und Technik der Ultrakurzen Wellen, by H. E. Hollmann. Erster Band—Erzeugung ultrakurzen Welliger Scgwingungen; Zweiter Band-die ultrakurzen Wellen in der Technik. (The Physics and Technique of Ultra-Short Waves)

Published (1936) by Julius Springer, Berlin, Germany; Edwards Brothers, Inc., (1943), Ann Arbor, Michigan. (Photo-Lithoprint Reproduction).

Volume I, 321 pages+5-page index+ix pages. 380 figures. Size, $5\frac{3}{4}\times8\frac{3}{4}$ inches.

Volume II, 300 pages+6-page index +viii pages. 283 figures. Size, $5\frac{3}{4}\times8\frac{3}{4}$ inches. Price, for the two volumes, $13.50, F.O.B. Ann Arbor, Michigan.

H. E. Hollmann's two volumes on "The Physics and Technique of Ultra-Short Waves" were originally published in 1936. The present publication is a photographic reproduction of this 1936 German edition. The work is now published under the auspices of the Alien Property Custodian presumably to make widely available to scientists and engineers certain useful material on microwaves. Similar considerations no doubt prompted the publication under the same auspices of the justly famous Jahnke-Emde "Tables of Functions"; but whereas a table of the error function, for example is never out of date, 1936 microwave techniques rather definitely are.

These books were initially reviewed in the December, 1936, issue of the Proceedings. The reviewer at that time confessed that in view of the mass of material presented he was unable to point out just how much of it would have permanent value. Today we are in a better position to make this judgment. Hollmann covers the entire field. Thus the chapter headings of volume I are: Spark Generation of Quasi-Optical Waves; The Generation of Ultra-Short Waves Using Feedback; The Retarding Field Method; The Magnetron, and the Generation of Ultra-Short Waves by Electron Beams. The volume closes with a typical German 295, title bibliography unfortunately arranged by chapters rather than on a topical basis. In volume II we have: Reception and Detection of Quasi-Optical Waves; Radiation and Formation of Beams; Propagation; Technical Applications of Ultra-Short Waves; and finally Measurements. A 338, title bibliography concludes this volume.

On the positive side we may say that this work represents one of the first attempts at a comprehensive survey of the entire field of short waves, with particular emphasis of frequencies above 300 megacycles. Since that time no other comparable survey has been published so the greatest value of these books today arises from their utility for purpose of reference. In certain fields technical progress has been very rapid in the past seven years. In such instances the treatment is inferior considered by present-day standards. In other fields which have been relatively unfruitful such as therapy, radio-frequency heating, and biology mentioned in volume II in the chapter on applications, the book may still be quite valuable and suggestive for workers in these fields.

The most serious defect of the work from the viewpoint of the research worker of today arises from the fact that many of the techniques described are obsolete. This situation is aggravated by the consistent emphasis on techniques rather than on fundamental theory. The books are descriptive rather than mathematical in character. Thus a great deal of material is included on generators of the retarding field type such as Barkhausen-Kurz oscillators. Today one is forced to look elsewhere for a treatment of the modern counterpart of the retarding field generator. The man who requires information on wave guides, cavity resonators, or velocity-modulation tubes will obviously not find his material in these books.

The books would be very valuable if brought up to date by discarding obsolete material and the inclusion of descriptions of the techniques developed in the last seven years. The reviewer does not expect such a book to be published until some years after the war.

W. D. Hershberger
RCA Laboratories
Princeton, New Jersey

Communication Circuits, by Lawrence A. Ware and Henry R. Reed

Published (1942) by John Wiley and Sons, Inc., 440 Fourth Ave., New York, N. Y. 281 pages+5-page index+vi pages. 122 illustrations. $6\frac{1}{4}\times 9\frac{1}{4}$ inches. Price, $3.50.

This is a good book. It does what it sets out to do. It gives a comprehensive analysis of the properties of transmission lines, filters, and some other associated circuits from the standpoint of the engineering student who has a reasonable mathematical background. It leads to the study of the properties of wave guides and coaxial cables, about one quarter of the book being devoted to their consideration.

Those practitioners of the circuit art who strive for incomprehensibility will undoubtedly turn up their patrician noses at the somewhat overdetailed algebra. From the standpoint of the elementary student however this detail is all to the good.

Greater care might profitably have been taken with the phraseology and to keeping before the students the physical processes involved. For instance on page 56 why say the $z^2/4$ term "somehow disappears"? Why not just say "disappears," there's nothing mysterious about it. Again on page 58 why say "the propagation constant per unit length which is taken here to be the mile"? This is confusing. As a matter of fact it isn't taken to be the mile or anything else. The formula is correct whatever units are used. Also on page 47 in discussing the bel and the neper why brand the logarithmic system to base e as being "more or less inconvenient"? It depends on what you are doing. If you are involved in mathematical theory logarithms to base e are generally the most convenient. However if you are interested in making arithmetical calculations, heaven help you if you have to use a table of logarithms to that base. The bel is defined in terms of logarithms to base 10 because man learned to count on his fingers and evolved a decimal scale of notation for his arithmetic.

These examples are given because they are typical of a certain looseness of phraseology which tends to mar the general excellence of the book. They should not be allowed to obscure the fact that the subject matter is generally well chosen and comprehensive. The work can be recommended not only as a textbook for the student but also as a reference volume for the engineer.

William Wilson
Pleasant Street
South Yarmouth, Massachusetts

Electronic Physics, by L. Grant Hector, Herbert S. Lein, and Clifford E. Scouten

Published by The Blakiston Co., 1012 Walnut St., Philadelphia, Pa. 346 pages+9-page index+viii pages. 298 figures. $6\times 8\frac{1}{2}$ inches. Price, $3.75.

Electronic Physics is written as a text for beginning students in electricity, light, and atomic physics. The material is presented, to quote the preface, " . . . from the electron-proton point of view. Even the ancient subject of magnetism is described in terms of moving electric charges. Ordinary light, wireless, and X-rays are shown to be closely related and all three to be various aspects of electrical phenomena." Such a presentation has interesting possibilities, and there can be no objection to the modernization of the teaching of elementary physics. However, the book is almost entirely qualitative and would not be of much value for engineering or science students, even for a first course. Other students would find that the brief mention of so many topics, ranging from simple frictional electricity to artificial radioactivity, in some three hundred pages, produced a mental confusion rather than a useful general picture.

The book is generously illustrated with sketches so designed as to emphasize the important features. One could wish for more photographs or drawings of actual equipment to help the student visualize the discussion. The text is quite readable and appears to be free of errors. For a book written in this country, the use of the term "wireless" instead of "radio" does not seem justified.

W. H. Pickering
California Institute of Technology
Pasadena, California

Mathematics Essential to Electricity and Radio, by Nelson M. Cooke and Joseph G. Orleans.

Published (1943) by McGraw-Hill Book Co., 330 West 42nd St., New York 18, N. Y. 374+36 pages+8-page index+x pages. 245 figures. $6\frac{1}{4}\times 9\frac{1}{4}$ inches. Price, $3.00.

This book was written to provide a mathematical background for students of electrical and radio engineering. It is an attempt "to adapt Lieutenant Cooke's book on Mathematics for Electricians and Radiomen to secondary-school level and to high-school teaching techniques."

Although primarily a book on mathematics, chapters are provided which deal with electrical and radio theory, so placed that each electrical section follows several of purely mathematical content. Thus, for example, several chapters on elementary algebra precede a section on Ohm's law in series circuits; chapters on the solution of equations precede a chapter on the use of Kirchhoff's laws for the calculation of currents in networks. This method of presentation, which has the advantage of sustaining the interest of students of electricity, leads logically to a chapter on the application of logarithms to decibels and transmission lines, and the material on trigonometry is illustrated, not only by exercises on the solution of triangles, but serves as a basis for the development of elementary alternating-current theory and the use of the complex and polar representation of alternating-current vectors.

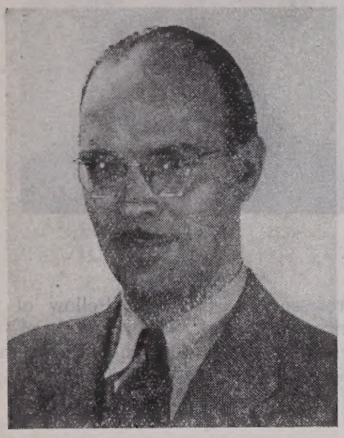
As a textbook on electrical engineering the book is naturally brief and somewhat superficial, but what is given is accurately and interestingly treated, and will be found eminently practical and useful for the man who has a working knowledge of electrical applications. The primary object of the book, to serve as a course in practical mathematics, is well accomplished, the method of presentation being clear and direct and well within the attainments of the student to which it is addressed.

An abundance of examples and problems is provided throughout the book, graded in difficulty so as to give facility in the use of the material in the text. The answers of the problems are printed at the end of the book, together with a wire-table, four-place logarithms, and natural trigonometric functions.

The book is beautifully printed and attractively bound.

Frederick W. Grover
Union College
Schenectady, N. Y.

Contributors



C. G. Brennecke

Cornelius G. Brennecke (A '36) was born in New York City on August 29, 1906. He received the degrees of A.B., B.S., and E.E. from Columbia University. His graduate work was done at New York University, where he received the degree of Ph.D. in 1936.

After several years with the engineering department of the Radio Corporation of America, Dr. Brennecke accepted a fellowship in physics at New York University, leaving there in 1936 to join the engineering faculty of the University of Toledo. Since 1942, he has been associate professor of electrical engineering at Lehigh University. He has published investigations in atomic physics and in the properties of dielectrics.

❖

Charles W. Harrison, Jr. (A '36) was born in Virginia in 1913. The years 1932–1936 were spent in the U. S. Naval Academy Preparatory School and in the Academy. In 1939 Mr. Harrison received the S.B. degree in engineering and in 1940 the degree of electrical engineer from the University of Virginia. In 1942 he was graduated with the S.M. degree in communication engineering from the Cruft Laboratory, Harvard University, and during the summer was a student at the Massachusetts Institute of Technology. His experience includes amateur, naval, and broadcast station operation, as well as research work at the Navy Department in Washington. He is at the present time a member of the teaching staff of the Cruft Laboratory.

C. W. Harrison, Jr.

❖

Rudolf G. E. Hutter was born in Berlin, Germany, on February 12, 1910. He was a graduate student in physics and mathematics at the University of Berlin from 1930 to 1935. ("State examination" 1936.)

From 1936 to 1938 he was a research physicist in the transmitter laboratories of Telefunken, G. m. b. H.; from 1939 to 1940, chief engineer of the radio station KZIB, Manila, Philippine Islands; from 1940 to 1941, he was a graduate student in communication engineering and physics at Stanford University, California. Since 1941 he has been research associate in the division of electron optics, Stanford University.

Mr. Hutter is a Member of Sigma Xi and of the American Physical Society.

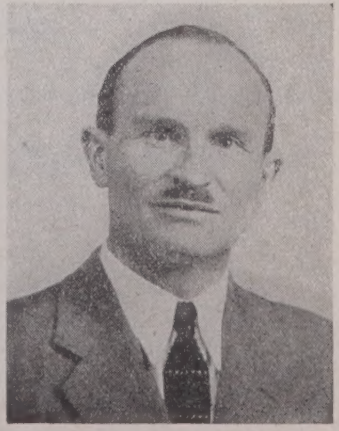
❖

Ronold King (A '30, SM '43) was born on September 19, 1905, at Williamstown, Massachusetts. He received the B.A. degree in 1927 and the M.S. degree in 1929 from the University of Rochester and the Ph.D. degree from the University of Wisconsin in 1932. He was an American-German exchange student at Munich from 1928 to 1929; a White Fellow in physics at Cornell University from 1929 to 1930; and a Fellow in electrical engineering at the University of Wisconsin from 1930 to 1932. He continued at Wisconsin as a research assistant from 1932 to 1934. From 1934 to 1936 he was an instructor in physics at Lafayette College, serving as an assistant professor in 1937. During 1937 and 1938 Dr. King was a Guggenheim Fellow at Berlin. In 1938 he became instructor in physics and communication engineering at Harvard University, advancing to assistant professor in 1939 and to associate professor in 1942.

R. G. E. Hutter

❖

William L. MacLean (A '42) was born on July 14, 1908 at Marinette, Wisconsin. He became interested in radio in grade school and operated a spark-coil—crystal-detector station in 1921. Later he operated two licensed amateur stations while in high school, and a third at Massachusetts Institute of Technology where he received the B.Sc. degree in electrical engineering in 1929. For the following year he worked in the Bell Telephone Laboratories. Thereafter, during the Weimar republic, he studied physics a semester each at the Universities of Munich and Berlin. This was followed by most of a third semester at the Sorbonne. He did actuarial work from 1932–1937. Mr. MacLean returned to electrical engineering



Ronold King

W. L. MacLean

in civil service from 1937 to 1941, and then went to the Polytechnic Institute of Brooklyn where he received the M.E.E. degree in 1942 and is now employed as a research associate. He is a member of Sigma Xi and of the Acoustical Society of America.

❖

L. Marton was born on August 15, 1901, in Budapest, Hungary. He received the Ph.D. degree in 1924 from the University of Zurich, Switzerland, where he remained as a research associate until 1925.

From 1926 to 1927 he was employed by the Tungsram Lamp Company as a research physicist. From 1928 to 1938 he was on the faculty of the University of Brussels, Belgium, where he begun his research in electron optics and microscopy; from 1938 to 1939 he was a lecturer at the University of Pennsylvania; from 1938 to 1941 he was a research physicist in charge of the development of electron microscopes, RCA Manufacturing Company; and visiting professor, summer session, University of Michigan, 1941.

Since 1941 he has been associate professor at Stanford University in charge of the division of electron optics.

L. Marton

Professor Marton is a Fellow of the American Physical Society and of the American Association for the Advancement of Science, a Member of Sigma Xi.